Relativistic length in high-energy physics

V. N. Strel'tsov
Joint Institute for Nuclear Research

Fiz. Elem. Chastits At. Yadra **22**, 1129–1170 (September–October 1991)

The role of the relativistic length in high-energy physics and its connection with the formation length is discussed. It is a consequence of their growth that large longitudinal distances become important at high energies. A relativistic long-range field effect is established on the basis of the Liénard–Wiechert potentials and the relativistic Yukawa potential. It is noted that the string and parton models of elementary particles and the hydrodynamic theory of multiparticle production are intimately related to or are implicitly based on the concept of the relativistic length. It is emphasized that the relativistic length also appears in interference experiments in which the spatial size of the region of generation of identical pions is measured in different frames of reference.

INTRODUCTION

In the middle of the sixties, a discussion arose around two questions in the theory of relativity.

One of them concerned the problem of equilibrium (the relativistic formulation of statics). This discussion touched on the standard formula for transformation of lengths. It was argued[1,2] that the longitudinal dimensions of relativistic objects could increase rather than be subject to the familiar Lorentz contraction. A part was also played here by the well-known problem of defining the energy and momentum of the electromagnetic field of a relativistic charge (the so-called 4/3 problem). Subsequently, this approach became known as the *asynchronous formulation*[3] (in contrast to the traditional *synchronous formulation*). In its framework, the spatial dimensions of relativistic bodies are defined by nonsimultaneous (asynchronous) positions of the elements that constitute them. However, this approach has not yet been recognized and is known to only a narrow circle of specialists. In the opinion of the present author, the main reason for this is that, in contrast to the standard (Einstein) definition, the asynchronous formulation is not based on a definite measurement procedure. Moreover, by itself it is in principle incapable of giving such a procedure without reference to a different (intrinsic) frame of reference of the object, and from the point of view of the relativity principle this is quite inadmissible.

Another of the questions touched on concerned the relativistic formulation of thermodynamics and, in particular, the formulas for transforming thermal energy and temperature. In accordance with the formula proposed by Ott,[4] the temperature of a moving body must increase, in contrast to the traditional ideas going back as far as Planck[5] and Einstein.[6] At the present time, there is no generally accepted description of relativistic thermodynamics, although the formulation adopted, for example, in Møller's well-known monograph on relativity theory[7] corresponds to the basic ideas of Ott.

Although these problems appear at the first glance completely independent, there is in fact a quite intimate connection between them, and this is expressed by the well-known equation of state of an ideal gas. Indeed, the requirement of Lorentz covariance of this equation and Ott's formula for the temperature lead uniquely to a relativistic expansion (and not contraction) of spatial volume.

The concept of the relativistic length was established and developed quite independently and alongside the discussion mentioned above, though at approximately the same time. In essence, the concept of the relativistic length is based on a nontraditional definition of the dimensions of rapidly moving objects, in particular their longitudinal dimensions. The measurement procedure employed is based on the well-known radar method of measuring distances. A consequence of the notion of the relativistic length is a relativistic "elongation formula." The intimate connection between the elongation formula and the relativistic "slowing down" of time can be regarded as a direct consequence of the unity of the space–time continuum, which, in its turn, is the main foundation of relativity theory.

It should be noted that the concept of the relativistic length has made it possible to resolve some well-known "paradoxes" of relativity theory and, indeed, eliminate the difficulties that arise in the attempt to make different branches of "classical physics" relativistic.

In the review now presented to the reader, we shall consider questions relating to the manifestation and use of the relativistic length in high-energy physics, in particular, the modification of this concept for application to elementary particles as composite objects of finite size.

We begin by considering the main aspects of the relativistic length itself, the connection between it and retarded distances in electrodynamics, its four-dimesional treatment, and modifications of the basic (radar) measurement procedure. Then, on the basis of the Liénard–Wiechert potentials, we consider the behavior of the field of a relativistic charge and trace the connection between the formation length of radiation and the relativistic length; we consider the well-known problem of defining the energy and momentum of the electromagnetic field of a relativistic charge. We shall show that a physical requirement—vanishing of the field momentum of a charge at rest—necessarily leads to a relativistic increase of spatial volume. On the basis of the expression for the relativistic Yukawa potential, we exhibit aspects of the behavior of the nuclear

0090-4759/91/050552-20$04.00

field of a nucleon; we discuss the way in which the duration of nuclear reactions and the formation lengths of hadrons change with increasing energy. We also consider some studies that posed the question of the role played by large longitudinal distances in the interaction of high-energy particles. We shall also consider the string and parton models of elementary particles and the hydrodynamic theory of multiparticle production. We shall consider the transformations of a relativistic electromagnetic dipole moment; we shall show that the absence of an intrinsic electric dipole moment of elementary particles leads to a stringent condition for the time coordinates of their constituents (this condition can also be used in the framework of the concept of the relativistic length). We also discuss the results of correlation experiments to measure the spatial size of the region of generation of identical pions in different frames of reference. For interactions of identical particles (or, at least, particles of the same mass) a manifestation of the elongation formula is expected here on the basis of a symmetry condition. Finally, it will be shown that a problem associated with the existence of a fundamental (elementary) length can only be solved from the standpoint of the relativistic length.

1. BASIC PROPOSITIONS OF THE CONCEPT OF THE RELATIVISTIC LENGTH

The generally accepted ideas about the behavior of the longitudinal dimensions of relativistic objects are based on Einstein's definition of length. In the framework of this definition,[8] the length of a moving (rigid) rod is the distance between the simultaneous positions of its ends. To obtain this, the observer establishes, using synchronous clocks at rest in a system S the points of the system S at which the beginning and end of the measured rod are situated at the time t. The distance between these two points is measured by applying a standard scale, and is called the length of the moving rod. It is well known that the Lorentz contraction formula is a consequence of this definition. It follows from it that all rapidly moving objects must undergo a contraction in the direction of their motion.

Definition of the relativistic length

The nontraditional definition of the relativistic length[9,10] is based on the radar method of measuring distances. In its framework, the length of a rod that is moving rapidly (for example, along its length) is defined as the half-sum of the distances traversed by light signals in the forward and backward directions along the rod, i.e., from one of its ends (A) to the other (B) and back. The procedure for measuring the propagation time of a light signal is identical to the corresponding procedure used to verify the formula for the relativistic retardation of time. In fact, on the basis of the latter we arrive at the elongation formula for longitudinal dimensions. However, we shall here give a different derivation of it.

For simplicity, suppose that the rod is oriented and moves in the direction of the x axis (from left to right) with speed $v = \beta c$. A signal is sent at the instant at which the left-hand end passes. The light reaches the right-hand end, is reflected there, and returns to the left end. For the distance traversed by the light signal as it moves in the same direction as the rod ("overtaking" the right end of the rod), we have

$$X_s = l^*(1+\beta)\gamma. \quad (1)$$

Here, l^* is the length of the rod at rest, and γ is the Lorentz factor $[\gamma = (1-\beta^2)^{-1/2}]$. When the light signal (after reflection) moves in the direction opposite to the direction of motion of the rod (toward the left end of the rod), it traverses the distance

$$X_0 = l^*(1-\beta)\gamma. \quad (2)$$

As a result, for the relativistic radar length we find

$$l_r = \frac{1}{2}(X_s + X_0) = l^*\gamma \text{ (elongation formula).} \quad (3)$$

We emphasize that X_s and X_0 are distances between points that are taken at different times, i.e., they obviously correspond exactly to the two most characteristic modifications of retarded distances in electrodynamics.

It is obvious that with increasing speed ($\beta \to 1$) the second distance tends to zero, $X_0 \to 0$, and the relativistic length will be essentially determined by half of X_s:

$$l_r \simeq \frac{1}{2} X_s. \quad (3a)$$

Retarded distances and the relativistic length

The concept of retarded distance was effectively introduced by Liénard and Wiechert.[11] In the simplest case in which the charge e moves along the x axis and approaches the point of observation,[1)] which also moves on the x axis, their formula for the electric potential φ has the form

$$\varphi = \frac{e}{X_r(1-\beta)}. \quad (4)$$

Here, X_r is the retarded distance, i.e., the distance between the point of observation (at the time t) and the charge (at the time t'), and $X_r = c(t - t')$; $v = \beta c$ is the speed of motion of the charge. In Minkowski space, the retarded distance is described by a segment of world line whose points are taken at different times.

In the rest frame of the charge, it is obvious that the expression (4) becomes the well-known expression for the Coulomb potential:

$$\varphi^* = e/X^*. \quad (5)$$

On the basis of (4) and (5), and using the transformation formula for the potential, we find that

$$X_r = X^*(1+\beta)\gamma. \quad (6)$$

In the opposite limiting case in which the field propagates in the direction opposite to the motion of the source,[2)] we have

$$\varphi = \frac{e}{X_r'(1+\beta)}, \quad (7)$$

and for the transformation formula of the corresponding retarded distance we find

$$X'_r = X^*(1-\beta)\gamma. \qquad (8)$$

Quite generally, one can say that it is precisely with retarded distances that relativistic electrodynamics operates. At the same time, the transition to the "instantaneous distance" (see, for example, Ref. 12), all of whose points correspond to the same instant of time, cannot be regarded as physically justified. The point is that by the time of observation t the charge may simply have "turned" from its path, having undergone some interaction after the time t'. Therefore, the instantaneous distance is in essence a fictitious quantity foreign to relativistic electrodynamics. Moreover, its introduction implicitly assumes the existence of an infinite interaction propagation velocity. Note that in everyday life radar measurements are exclusively concerned with retarded distances. Nevertheless, Einstein's definition of the length of a moving rod, which is used to explain the Lorentz contraction, is actually based on the concept of the instantaneous distance. In contrast, in the framework of the alternative definition (the relativistic length) it is retarded distances that are used. Indeed, the expressions (6) and (8) correspond exactly to the expressions (1) and (2).[3)] Thus, one can say that the concept of the relativistic length is an organic consequence of relativistic electrodynamics. Indeed, it cannot be otherwise, since the concept is based on the radar method of distance measurement.

Apparent size of rapidly moving objects

In the considered radar method of measurement of the relativistic length, the light signal moving in the direction of the rod "overtakes" its right end. In the opposite direction, the left end of the rod moves toward the light signal. We actually have a similar situation when we follow the change in the apparent size of rapidly moving bodies. Strictly speaking, it is assumed here that the longitudinal dimension is seen by a point observer. When we say "seen," this means that the observer simultaneously notes signals that were emitted at different times, say by the ends of the rod. Such a mechanism of "seeing" is indeed confirmed by the results of direct experiments of Duguay on photographing light in flight.[13] In practice, an observer situated, for example, near the path of motion of the rod will see an approaching rod elongated by $(1+\beta)\gamma$ times and a departing rod shortened by $(1-\beta)\gamma$ times, etc.[10,14] The mean apparent length will be determined precisely by the elongation formula (3). It should be mentioned here that in this problem there is a point (or, rather, a plane) of symmetry, when the middle of the rod is at the minimal distance from the observer. In this position, the apparent longitudinal dimension is precisely l_r. However, in our opinion the most important thing here is that for every position of the center of the rod to the left of the observer there corresponds a "symmetric" position to the right. Moreover, the mean value of these longitudinal dimensions will always be l_r. For example, for the Lorentz-contracted length $l^*\gamma^{-1}$ (in the "right" position), we have the corresponding "left" value $(1+\beta^2)l^*\gamma$. It may be worth making a special mention here of the investigations into the behavior of the apparent shape of a rapidly moving sphere,[15] from which, in particular, it follows that the famous Lorentz-contracted disk is simply unobservable.[4)]

A detailed discussion of the relativistic effects of the visual perception of the shape of moving bodies, which do not reduce to Lorentz contraction, can be found in a paper of Bolotovskiĭ.[17] The important thing here is that the processes of perception or photographing of moving objects are essentially a modification of the radar method of measurement.

In addition, it is particularly important for what follows that the process of perception is associated with the interaction of emitted light signals (ultimately photons) with an observer or a detecting device. In other words, the "perceived" dimensions must reflect the actual nature of the interaction (in the given case, electromagnetic). Quite generally, we can say that in accordance with modern ideas the mechanisms of both electromagnetic and strong interactions actually have their basis in radar location (or "perception" by means of photons and gluons, respectively).

Four-dimensional representation

In the framework of the four-dimensional representation, the relativistic length can be expressed by the spatial part of the half-difference of the two 4-vectors x^i $(i=0,1,2,3)$ that describe the processes of propagation of light in the forward (X_{AB}) and backward (X_{BA}) directions along the rod. In the rest frame S^* of the rod, we have

$$X^{i*}_{AB}(l^*/c, l^*, 0, 0), \qquad (9a)$$

$$X^{i*}_{BA}(l^*/c, -l^*, 0, 0). \qquad (9b)$$

On the basis of special Lorentz transformations, we find for the system S in which the rod moves

$$X^i_{AB}[(1+\beta)l^*\gamma/c, (1+\delta)l^*\gamma, 0, 0], \qquad (10a)$$

$$X^i_{BA}[(1-\beta)l^*\gamma/c, -(1-\beta)l^*\gamma, 0, 0]. \qquad (10b)$$

As a result, for $l^i_r = (X^i_{AB} - X^i_{BA})/2$ we have

$$l^{i*}_r(0, l^*, 0, 0), \qquad (11)$$

$$l^i_r(\beta l^*\gamma, l^*, 0, 0). \qquad (12)$$

However, the expression (11) actually indicates that the relativistic length l_r can also be obtained if, for example, one uses for its determination sources that are situated at the ends of the rod and simultaneously (from the point of view of S^*) emit signals. Thus, we have here another modification of the definition of the relativistic length. However, it is obvious that for this we need an *a priori* meaning of the statement that the sources "emit" simultaneously in S^*.

As we have already noted, an approach associated with the use of the expression (3) as the transformation formula for the (longitudinal) length was discussed in the sixties by several authors (among the papers, we mention Refs. 1, 2,

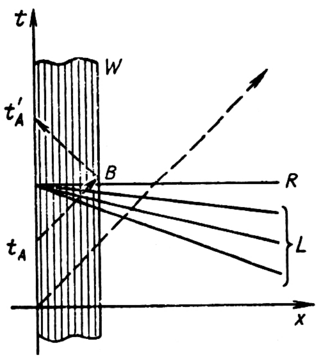

FIG. 1. World strip of a rod.

and 18), especially in the framework of the so-called asynchronous formulation[5)] (Refs. 3, 19, and 20). However, whereas the standard definition is based on Einstein's specific procedure for measuring the length of a moving rod,[8] in the cited papers (except for Refs. 3 and 19) a procedure for measurement of the quantities occurring in the expression (3) was not considered at all. Moreover, as we have already said, the asynchronous formulation by itself is in principle incapable of giving a measurement procedure in S without reference to another system S^*, and this gives rise to a feeling of dissatisfaction.

However, in any consistent physical theory, a given quantity can be regarded as defined only if definite operations by means of which this quantity can be measured are indicated. The procedure for measuring the relativistic length, based on the direct use of clocks and light signals, gives just such a prescription for the determination of l^* and l_r. Only after this does the mathematical expression (3) acquire physical meaning.

As regards the geometrical representation of a rod, at the first glance this seems a fairly simple matter. However, relativity theory established that physically, a material ruler (rod) is not a spatial object but a space–time configuration. This two-dimensional configuration is a space-like world strip in four-dimensional Minkowski space. In the intrinsic frame of reference S^*, in which the rod is at rest, its world lines (which form the strip) are parallel to the time axis t. In the simplest case of flat Minkowski space, represented in Fig. 1 in Cartesian coordinate systems, the world strip of the rod is vertical. The 4-vectors (in the given case, 2-vectors) describing the processes of light propagation in the forward and backward directions along the rod are represented by the lines t_AB and Bt'_A, respectively; t_A and t'_A are the times of despatch and reception of the light signal. In accordance with (11), the relativistic length obviously corresponds to the normal section of S with the world strip of the rod.[6)] Thus, there exists a simple connection between the mutual disposition of the direction of the world strip W and the line R, which is the locus of the events that relative to W satisfy the definition of light simultaneity (Ref. 21): In any frame of reference, the world lines of W and the line R make the same Euclidean angles with the world line of a light signal.

However, if the normal section R depends only on the actual world strip W of the rod, and not on the choice of the frame of reference, this means that the defintiion of the relativistic length satisfies the requirement of Lorentz covariance. Moreover, we have here complete analogy with the definition of relativistic time. On the other hand, to the traditional definition of the length of a rapidly moving rod there will correspond a whole set of sections L, each of which is determined by its "own" frame of reference.

Volume of a rapidly moving object

The question of the volume of a relativistic body can be answered on the basis of the following simple thought experiment.

For this, we consider a sphere at rest (in the system S^*) of radius l^* (with specular internal surface), at the center of which there is a light source. At the time $t^* = 0$, this source emits a spherical wave, the front of which reaches the surface of the sphere after the time l^*/c. At time $\Delta t^* = 2l^*/c$ after emission, the reflected wave reconverges at the center of the sphere O^*. According to observations in the system S, in which the sphere moves, the points of emission and absorption will be separated by a distance $OO' = 2\beta l^*\gamma$. Since at the same time the half-sum of the paths there and back will be the same for each ray and equal to $l^*\gamma$, it is obvious that in accordance with the observations in S the front (corresponding to the time $t^* = l^*/c$) will have the shape of an ellipsoid of revolution prolate along the x axis with semiaxes $a_y = a_z = l^*$ and $a_x = l^*\gamma$. The focal parameter will be $p = a_y^2/a_x = l^*\gamma^{-1}$, the focal distance $f = \sqrt{a_x^2 - a_y^2} = \beta l^*\gamma^{-1}$, and the eccentricity $e = f/a_x = \beta$, and the two quantities considered above will be $X_s = a_x + f$ and $X_0 = a_x - f$. Now, using
the formula $V = 4\pi a_x v_y^2/3$ for the volume of an ellipsoid, we find that as a result of the motion the volume has been increased by γ times:

$$V = V^*\gamma, \tag{13}$$

where $V^* = 4\pi l^{*3}/3$ is the volume of the sphere.

2. FIELD OF A RELATIVISTIC CHARGE. THE FORMATION LENGTH

The Liénard–Wiechert potentials

The general relativistically covariant expression for these potentials has the form

$$A^i = \frac{eu^i}{R_k u^k}. \tag{14}$$

Here, u^i is the 4-velocity of the charge; R^k is the 4-vector of the retarded distance $R^k = [c(t - t'), \mathbf{R} - \mathbf{R}']$; $X^{i'}$ are the coordinates of the charge; X^i are the coordinates of the point of observation, and R^k is a lightlike "null vector," i.e.,

$$R_k R^k = 0. \tag{15}$$

The formula for the electric Liénard–Wiechert potential φ that follows from (14) in polar coordinates is

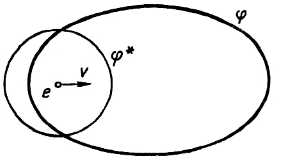

FIG. 2. Liénard–Wiechert equipotential ellipse, $v = 0.75c$. The circle represents the Coulomb equipotential.

$$\varphi = \frac{e}{R_r(1-\beta\cos\theta)}, \tag{14a}$$

where R_r is the distance from the point at which the charge is situated to the point of observation. On the basis of (14a) it is possible to construct equipotential curves for the relativistic charge. They will obviously be determined by the equation

$$R_r = \frac{e/\varphi}{1-\beta\cos\theta}, \tag{16}$$

which is the polar equation of an ellipse; e/φ is the focal parameter, and β is the eccentricity of the ellipse. Such a curve is shown in Fig. 2 for $\beta = 0.75$ ($\gamma = 1.5$); the circle corresponds to the Coulomb potential (charge at rest). As can be seen from the figure and as follows from the expression (16), the field of the charge is drawn out forward as its velocity is increased and acts at ever greater distances.[22] This behavior of the field is actually determined by the retardation factor $\varkappa = 1 - \beta\mathbf{n}_r$, where $\mathbf{n}_r = \mathbf{R}_r/R_r$. In particular, it is easy to see that

$$X_r \approx 2\gamma^2. \tag{17}$$

Thus, one can say that there is a kind of relativistic long-range effect. We note also that at the velocities in which we are interested ($\beta \approx 1$), the component A^1 of the field, for example, will obviously behave similarly. It is important to emphasize that the longitudinal dimension of the field is essentially specified by the characteristic retarded distance when the source catches up with its own field, this corresponding to the quantity X_s, which effectively determines the relativistic length.

The following arguments will be purely qualitative in nature and will address the question of the maximal distance from a relativistic charge at which a certain test charge has an energy not less than a given value ($\mathscr{E}$). It is obvious that the formula (16) enables us to obtain such an estimate. Indeed, it follows from (16) that

$$(R_r^m)^{-1} \approx e\varphi(1-\beta\cos\theta) = \mathscr{E}(1-\beta\cos\theta). \tag{16a}$$

The last expression can obviously also be used to estimate the energy of an electron–positron test pair. However, this pair, in its turn, can, in particular, be the result of transformation of a photon radiated by the relativistic charge itself. Thus, in our opinion, the expression (16a) can actually serve for obtaining qualitative estimates of the size of the region (zone) of radiation of a given energy (frequency).

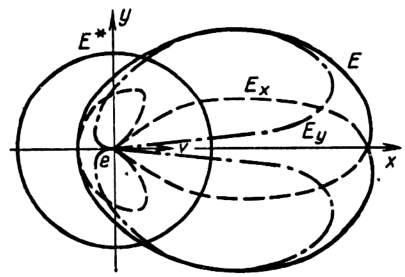

FIG. 3. Contours of equal strength of the field and the field components of a moving charge, $v = 0.75c$. The circle corresponds to the charge at rest.

"Retarded field strengths"

In this subsection we shall consider how the field, or, rather, the lines of equal field strengths, of a relativistic charge behave when its velocity is changed. At the same time, we shall briefly consider the possible formation of a bound system of two relativistic charges (for example, positronium) through the long-range effect, i.e., at large distances between the charges.

To address our first question, we use the well-known formula for the strength of the electric field[7)] produced by a point charge moving with constant velocity (see, for example, Ref. 12):

$$\mathbf{E} = \frac{e\gamma^{-2}}{(R-\mathbf{R}\boldsymbol{\beta})^3}(\mathbf{R}-\boldsymbol{\beta}R). \tag{18}$$

Here, $\mathbf{v} = \boldsymbol{\beta}c$ is the velocity of the charge e, and $\mathbf{R}$ is the vector of the (retarded) distance. For simplicity, we shall take $\boldsymbol{\beta}$ (β, 0, 0) and restrict ourselves to consideration of the xy plane. At the same time, we rewrite (18) in components in the form

$$E_x = \frac{e\gamma^{-2}}{R^2(1-\beta\cos\theta)^3}(\cos\theta-\beta),$$
$$E_y = \frac{e\gamma^{-2}}{R^2(1-\beta\cos\theta)^3}\sin\theta. \tag{18a}$$

To establish the lines of equal field strengths themselves, we need the formula for $E = |E|$ that follows from (18a). It is readily seen that it is

$$E = \frac{e\gamma^{-2}}{R^2(1-\beta\cos\theta)^3}(1+\beta^2-2\beta\cos\theta)^{1/2}. \tag{19}$$

The results of calculations of the equal-strength curves are shown in Figs. 3 and 4. In the first case $v = 0.75c$, $\gamma = 1.5$; the curves for E, E_x, and E_y were obtained on the basis of (19) and (18a), respectively. The E curve goes over into the circle E^* in the limit $v \to 0$. It can be seen that at $\theta = \cos^{-1}\beta$ the component of the field strength vanishes; it then changes sign (becoming negative). Note that the "range" of the field of the moving charge is increased. For the mean value we have $\langle R\rangle = 1.15$, and $R_\perp^{\max} = 1.24$ (the radius of the circle E^* is taken as unity). As can be seen from Fig. 4, if the velocity of the charge is

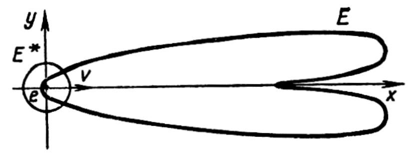

FIG. 4. Contours of equal field strengths of a charge at rest (circle) and in motion, $v = 0.98c$.

increased ($\gamma = 5$), the range of the field is also increased, $\langle R \rangle = 1.8$. The field is drawn out forward more and more. It is interesting to note that this effect is well known and appears in the calculation of the angular distribution of the radiation field when the second acceleration-dependent term [omitted in (17)] is taken into account (see, for example, Ref. 23). Experiments investigating the behavior of radiation can in fact be regarded as evidence that the distance (length) in a moving system is determined by nonsimultaneous positions of points (of the charge and point of observation).

We now consider the following example. As a moving charge we take an electron and as a test charge, on which the field will act, a positron. Suppose first that the electron moves with very low velocity in the direction toward the positron. When the distance between them is about 10^{-8} cm, the electron and positron can form a bound system—positronium. Using a different language, one can say that positronium is formed when the field of the electron at the position of the test charge, the positron, reaches a certain given value. We then consider a relativistic electron. With allowance for the long-range field effect, a much greater distance between the electron and positron will correspond to the field value that usually results in formation of a bound system.[8)] In other words, the maximal longitudinal dimension of the relativistic positronium formed in this manner will be appreciably greater than the longitudinal dimension at rest. With increasing energy, their ratio must increase in proportion to the Lorentz factor γ. Thus, at energy $\mathscr{E}_p \approx 10^2$ GeV the dimension will be about 10 μm, i.e., essentially a macroscopic quantity. In our opinion, it is this phenomenon that is the basis of the growth of the formation length of relativistic particles and, in particular, positronium when the energy is increased. It appears entirely natural that for a more extended object there will be a greater path of its formation. On the other hand, the widely accepted belief in the Lorentz contraction of relativistic objects is difficult to reconcile with growth of their formation lengths. Indeed, in the case of positronium the maximal longitudinal dimension at the same energy $\mathscr{E}_p \approx 10^2$ GeV is about 10^{-13} cm, corresponding already to nuclear dimensions. How can one then explain such large formation lengths of relativistic positronium?[24]

Formation lengths

It is well known that many electromagnetic processes that take place when fast charged particles interact with matter unfold in a large spatial region along the direction of the particle momenta. The length of this region, which is called the coherence length or formation length of the radiation, increases strongly with increasing energy of the particles.

The concept of formation length was introduced by Frank[25] when considering the radiation of a uniformly moving oscillator in a refractive medium. He defined it as the interval of the path from which all waves are radiated by the oscillator (source) in phase.

It was found subsequently that the formation length plays an important part in radiation processes of fast particles (see, for example, Refs. 26 and 27). The value of this length enables one not only to obtain a qualitative picture of the radiation but also to estimate with good accuracy quantitative characteristics of the radiation such as the spectrum, angular distribution, total losses, etc. On the other hand, the growth of the formation length with increasing energy suggests that there is a possible connection between it and the relativistic length.

We consider the motion of a fast charged particle along a straight line. At each point of its path, the particle radiates a plane wave of frequency ω with wave vector k. The phase difference $\Delta\varphi$ of the waves radiated at angle θ to its momentum at the times t and $t + l/v$ will be

$$\Delta\varphi = \omega \frac{l}{v} - kl \cos \theta, \tag{20}$$

where l is the path traversed, and v is its velocity. The formation length l_f is defined as the distance for which $\Delta\varphi = 1$. It is then readily seen that

$$l_f = \frac{v}{\omega(1 - \beta \cos \theta)}. \tag{21}$$

Thus, by definition l_f is the length of the path from which waves reach the point of observation with phase difference not greater than π, i.e., from the wave point of view simultaneously. At the same time, l_f is a numerical measure of the spectral and angular distribution of the radiation. These characteristics of the radiation are proportional to the square of the formation length.

We also note that the expression (21) can be obtained by regarding the formation length as the distance over which the photon is "separated" from the charge that radiates it.

Since at high energies $v \approx c$, it follows from (21) that

$$l_l^{-1} \approx \omega(1 - \beta \cos \theta). \tag{21a}$$

It can be seen that at high energies the formation length will be essentially determined by the retardation factor $\varkappa$, i.e., like R_r^m (16a), which describes the size of the field of a relativistic charge. Thus, it can be said that the formation length and the relativistic length have a behavior of the same nature. Moreover, it appears that it is precisely the relativistic long-range field effect that actually determines the growth of the formation length at high energies.

Our previous treatment was purely classical, i.e., we assumed that the charged particle moves uniformly, and we did not take into account, for example, the recoil resulting from radiation of a photon. In quantum theory, a quantity corresponding to the formation length is introduced on the basis of a matrix element[28] that contains in an integral over the spatial coordinates a factor $\exp(i\mathbf{qr})$, where $\mathbf{q}$ is the variable momentum, $\mathbf{q} = \mathbf{p}_0 - \mathbf{p} - \mathbf{k}$, $\mathbf{p}_0$ and $\mathbf{p}$ are the momenta of the charge before and after radiation, $\mathbf{k}$ is the momentum of the radiated photon, and $\hbar = 1$.

The exponential determines the effective value of r that makes the main contribution to the matrix element. At high energies, the radiation process unfolds along the direction of the particle momentum, and important contributions are therefore made by

$$q_{\parallel} = p_0 - p - k \approx \frac{\omega m^2 c^3}{2\mathscr{E}_0 \mathscr{E}}, \tag{22}$$

where $\mathscr{E}_0$ and $\mathscr{E}$ are the energy of the electron before and after radiation, and m is its mass. It follows from this that the photon formation length is defined as

$$l_f = \frac{2\mathscr{E}_0\mathscr{E}}{m^2 \omega c^3}. \tag{23}$$

Energy and momentum of the electromagnetic field of a charge

As is well known, the 4-momentum of the electromagnetic field is determined by the integral

$$G^i = \int T^{ik} dV_k, \tag{24}$$

where T^{ik} is the energy–momentum tensor of the electromagnetic field,

$$T^{ik} = -F^{il}F_l^k + \frac{1}{4}\gamma^{ik}F_{mn}F^{mn}, \tag{25}$$

F^{ik} is the field-strength tensor of the electromagnetic field, and dV_k is a 4-vector of an infinitesimal volume:

$$dV_k = -\varepsilon_{klmn} dx^l \delta x^m \Delta x^n. \tag{26}$$

Here, ε_{klmn} is the Levi-Civita pseudotensor ($\varepsilon_{0123} = -1$). In particular, dV_k can have the form

$$dV_k(dx^1\delta x^2 \Delta x^3, -dx^0\delta x^2\Delta x^3, -dx^1\delta x^0 \Delta x^3, -dx^1 \delta x^2 \Delta x^0), \tag{26a}$$

where dV_0 is obviously the element of ordinary spatial volume ($dV_0 \equiv dV$).

Many years ago, in connection with Abraham's hypothesis of the electromagnetic origin of the electron mass, the following expressions were obtained for the momentum and energy (see, for example, Ref. 29):

$$G^1 = (4/3)m\beta c\gamma, \quad \mathscr{E} = mc^2(1+\beta^2/3)\gamma, \tag{27}$$

where $m = \mathscr{E}^*/c^2$, $\mathscr{E}^*$ is the electrostatic energy in the rest frame of the electron, and $\beta c = v_x$ is the velocity of the electron's motion. It should be noted that these expressions were derived using, in particular, the Lorentz formula for volume contraction.

It is obvious that the expressions (27) obtained in this manner differ significantly from the well-known relativistic expressions

$$p^1 = m\beta c\gamma, \quad \mathscr{E} = mc^2\gamma \tag{28}$$

for the momenta and energy of a moving mechanical particle with rest mass m.

Below, we shall discuss this problem in detail. As usual, we shall first consider a given charge in the intrinsic frame of reference (S^*), in which it is at rest ($G^* = 0$). Since for a charge at rest the magnetic field vanishes, $\mathbf{H}^* = 0$, and $F_*^{ik} = (-\mathbf{E}^*, 0)$, the components T_*^{ik} will be given by

$$T_*^{ik} = \begin{pmatrix} \frac{1}{2}(\mathbf{E}^*)^2 & 0 & 0 & 0 \\ 0 & -(E_x^*)^2 + \frac{1}{2}(\mathbf{E}^*)^2 & -E_x^*E_y^* & -E_x^*E_z^* \\ 0 & -E_x^*E_y^* & -(E_y^*)^2 + \frac{1}{2}(\mathbf{E}^*)^2 & -E_y^*E_z^* \\ 0 & -E_x^*E_y^* & -E_y^*E_z^* & -(E_z^*)^2 + \frac{1}{2}(\mathbf{E}^*)^2 \end{pmatrix}. \tag{29}$$

Using the condition of spherical symmetry of the field, we obtain

$$\int E_*^{\mu} E_*^{\nu} dV^* = \frac{\delta^{\mu\nu}}{3} \int (E^*)^2 dV^*, \tag{30}$$

and also

$$\int E_{*\mu} E_*^{\nu} V_{\nu}^* = 0 \quad (\mu,\nu = 1,2,3). \tag{31}$$

In the system S^*, the charge is at rest, and therefore we must require vanishing of the components of the momentum $\mathbf{G}^*$:

$$G_*^1 = \frac{1}{6} \int (E^*)^2 dV_1^*$$

$$= 0, \quad G^2_*$$

$$= \frac{1}{6} \int (E^*)^2 dV^*_2$$

$$= 0,$$

$$G^3_* = \frac{1}{6} \int (E^*)^2 dV^*_3 = 0. \qquad (32)$$

Since the integrands here are essentially positive, vanishing of the integrals (32) will be possible only if[30]

$$dV^*_1 = dV^*_2 = dV^*_3 = 0. \qquad (33)$$

The last condition will be automatically satisfied if in the system S^* we define the 4-vector of the volume element (26a) by means of three 4-vectors of the following form:

$$dx^i_*(0,dx^*,0,0), \quad \delta x^i_*(0,0,dy^*,0), \quad \Delta x^i_*(0,0,0,dz^*). \qquad (33a)$$

The physical meaning of this choice of the vectors dx^i_*, δx^i_*, and Δx^i_* becomes clear if we consider the procedure for measuring spatial intervals (lengths) by the radar location method. It is then obvious that each of these vectors can be represented as the half-difference of two "lightlike" 4-vectors describing the processes of propagation of a light signal along the corresponding infinitesimal spatial interval in the forward and backward directions. In other words, such a choice of the vectors dx^i, δx^i, and Δx^i corresponds precisely to the definition of the relativistic length considered above and, in particular, the expression (11).

Using special Lorentz transformations for transition to a system S that moves along the x^* axis of the system S^* with velocity $v_x = -\beta c$, we obtain on the basis of (26a) and (33a) the formulas for transforming the components dV_i:

$$dV = dV^*\gamma, \qquad (34a)$$

$$dV_1 = -\rho dV^*\gamma, \quad dV_2 = dV^*_2 = 0, \quad dV_3 = dV^*_3 = 0. \qquad (34b)$$

In the considered special case, the energy and momentum of the moving charge will be determined by the expressions

$$\mathscr{E} = \int T^{00} dV_0 + \int T^{01} dV_1; \qquad (35a)$$

$$G = \frac{1}{c}\left(\int T^{10} dV_0 + \int T^{11} dV_1 \right). \qquad (35b)$$

Using further the transformation formulas for the components of the energy–momentum tensor T^{ik},

$$T^{00} = (T^{00}_* + \beta^2 T^{11}_*)\gamma^2; \qquad (36a)$$

$$T^{01} = T^{10} = \beta(T^{00}_* + T^{11}_*)\gamma^2; \qquad (36b)$$

$$T^{11} = (T^{11}_* + \beta^2 T^{00}_*)\gamma^2, \qquad (36c)$$

and taking into account (34), we readily find

$$\mathscr{E} = \gamma \int T^{00}_* dV^* = \mathscr{E}^*\gamma, \qquad (37a)$$

$$G^1 = \frac{1}{c}\beta\gamma \int T^{00}_* dV^* = \frac{1}{c}\beta\mathscr{E}^*\gamma. \qquad (37b)$$

It is obvious that the formulas (37) correspond to the usual relativistic transformation formulas for the momentum and energy (28) and differ from the well-known expressions (27). Thus, in the framework of this approach there is no need to ascribe to the electron additional mechanical inertial mass due, say, to the existence of nonelectric forces—"Poincaré stresses" (Ref. 31).

The covariant definition of electromagnetic momentum and energy and the associated derivation of the expressions (37) have been considered by several authors (see, for example, Refs. 2 and 32). However, it must be emphasized that the requirement of covariance alone is not sufficient to obtain the expressions (37), since, for example, the well-known expressions (27) also satisfy the requirement if it is borne in mind that in this case $G^1_* \neq 0$.

Indeed, if the 4-vector dV_i in the system S is chosen in the form $(0,dV,0,0)$, ensuring the Lorentz contraction, this means that in the system S we have $(\beta dV\gamma, dV\gamma, 0,0)$ or $(\beta dV^*, dV^*,0,0)$. As a result, we find that $G^1_* = \beta c\mathscr{E}^*/3$, and the corresponding quantities will indeed be related by a Lorentz transformation, as is readily verified.

In connection with what has been said, we want to say something about the paper of Gamba,[18] which, in particular, criticizes the standard procedure for calculating the energy and momentum of the electromagentic field of a charge in different frames of reference (S and S^*) on the basis of integration over spatial volumes at $t =$ const and $t^* =$ const, respectively. Since the integration is, thus, over different hypersurfaces, the results of the calculations must, as the author notes, correspond to different sets of physical events, whereas Lorentz transformations deal with the same set of events.

Now as regards the choice of a (spacelike) surface of integration in the calculation of the integral (24), *a priori* it would indeed appear to be difficult to speak of any preferred surface.[33] However, it must here be borne in mind that in all cases except integration over surfaces orthogonal to the world lines of the charge the momentum of a charge at rest is nonzero. This fact leads us to a physical condition, and the requirement of fulfillment of this condition (vanishing of the momentum of a charge at rest) uniquely determines the choice of the surface of integration.

Thus, relativistic electrodynamics (by imposing stringent requirements on the choice of the surface) in fact unambiguously indicates, in accordance with (34a), that there is an increase (and not contraction) of a moving volume.

3. RELATIVISTIC NUCLEON. DURATION OF NUCLEAR REACTIONS

According to modern ideas, the basic features of the strong interactions are described by quantum chromodynamics. In particular, the gluon field is in a definite manner analogous to the electromagnetic field. At the same time, this theory naturally includes earlier results that explain, for example, the short range of nuclear forces. In the early days of nuclear physics, an important role was played by Yukawa's idea,[34] according to which nucleons interact through the exchange of π mesons. The potential of the Yukawa interaction (for a stationary meson field) has the form

$$\varphi_\pi = -g_\pi \frac{\exp(-\mu R)}{R}. \tag{38}$$

Here, g_π is a coupling constant analogous to the electron charge in electrodynamics, and μ is the mass of the π meson; as a rule, we shall set below $c = 1$. Later, to explain the behavior of the strong interactions at higher energies, it was also necessary to take into account the exchange of heavier (vector) mesons: ρ, ω, etc.

It is the presence of the Yukawa exponential of the function φ_π that makes the "range" of the nuclear forces much less, by a factor of about μ^{-1} (~ 1 fm), than the "range" of the electrostatic forces described by the Coulomb potential. In other words, one can say that for photons, as for gluons, we simply have $\mu_{\text{ph}} = \mu_g = 0$.

Relativistic Yukawa potential

In view of the analogy between the electromagnetic and gluon fields, the results of Sec. 2 can also be used to a certain extent for qualitative explanation of the behavior of strong interactions at high energies. However, since "at their limit" hadronization occurs, for example, the transformation of quarks into virtual pions (or other mesons), it nevertheless appears more correct to proceed from the Yukawa potential. For a moving nucleon, the corresponding expression for the potential can be obtained by using Lorentz transformations. Comparison of (38) with the expression (39a) given below, which describes a special case, provides a transparent illustration of this. In the general case, the manifestly relativistically invariant expression for the Yukawa potential will have the form[22]

$$\varphi_\pi = -g_\pi \frac{\exp(-\mu u^i R_i)}{u^i R_i}. \tag{39}$$

Here, u^i is the 4-velocity of the nucleon, and R_i is the 4-distance from the point of "observation" (P) to the "charge" (center of mass of the nucleon). In the simplest case, when the point P lies on the x axis and the nucleon approaches it along the same axis, the expression (39) becomes

$$\varphi_\pi = -g_\pi \frac{\exp[-\mu R_\parallel (1-\beta)\gamma]}{R_\parallel (1-\beta)\gamma}. \tag{39a}$$

Here, $R_\parallel$ is the retarded distance, and β is the velocity of the nucleon.

As we have already shown above, the electric field ahead of a moving charge increases as its velocity is increased in accordance with the expression (17). It follows directly from it that at a given distance in front of a moving charge the potential of the field is $\sim 2\gamma^2$ times greater than at the same distance from a charge at rest. Because of the presence of γ in the denominator of the pion potential (39a), this growth will occur much more weakly in the given case. If $\varphi \sim \gamma^2$, then $\varphi_\pi \sim \gamma$.

In the case of the vector ρ and ω mesons, the expression for the potential corresponding to (39) has the form

$$\varphi_V = -\sum_{\rho,\omega} g_V \frac{\exp[-\mu_V(1-\beta\cos\theta)\gamma]}{R(1-\beta\cos\theta)}. \tag{40}$$

Since at high velocities, in which we are interested, $\beta\gamma \approx \gamma$, the x component of the potential will behave in practically the same way as the time component φ_V.

Because the masses of the ρ and ω mesons are similar, the exponential can be taken approximately in front of the summation sign. Then for the vector field we have $q_V = g_\rho + g_\omega$.

Shape of a relativistic nucleon

The problem of the shape of a rapidly moving nucleon reduces essentially to finding the equipotential surfaces that describe the behavior of the range of its nuclear field. The point of departure here is the Yukawa potential at the distance of the pion Compton wavelength $\varphi_\pi(\mu^{-1}) = [\varphi]$, which obviously corresponds to the usual definition of the range of the nuclear forces. On the basis of this condition, a search was made for solutions of Eqs. (39) and (40) for R, i.e., R was simply chosen in such a way as to reduce the value of the potential φ_V to $[\varphi]$.

Since the pion potential is a scalar quantity with respect to Lorentz transformations, the problem actually reduces in this case to finding the shape of the surface of the front of a light wave propagating from a moving source under the condition that in the rest frame (S^*) of the source this wave has a spherical shape. But this problem has already been considered above in Sec. 1. It was shown that the required surface has the shape of an ellipsoid of revolution prolate in the direction of motion with semiaxes $a_x = R_\pi^*\gamma$ and $a_y = R_\pi^*$ and focal distance $f = \beta R_\pi^*\gamma$, where R_π^* is the radius of the sphere in the system S^*. It follows from this that the range of the pion field in the forward direction is $R_\parallel^f = a_x + f$, and that in the backward direction is $R_\parallel^b = a_x - f$.

The results of calculations[35] in which it was assumed that $g_V = 0.77$ (Ref. 36) are presented in three figures and in Table I. Figure 5 shows the field of a comparatively "slow" nucleon, although, as can be seen in the case of Fig. 5b, $\beta = 0.98$. Also shown here are the corresponding equipotential curves (having the shape of a circle) for a nucleon at rest. It follows from comparison of Figs. 5 and 6 that with increasing γ the nuclear field of the nucleon is drawn forward more and more and acts at ever greater distances. For this reason, the scale in Fig. 6 along the x axis has been changed. It can be seen that the range of the vector field increases more rapidly and at $\gamma = 200$ already clearly exceeds the range of the pion field. It is particularly important that the transverse dimensions of the field begin to increase. But since the considered fields essentially rep-

TABLE I. Dependence of the longitudinal and transverse dimensions of the nuclear field of the nucleon on the Lorentz factor.

γ	π field			ρ, ω fields		
	$R^f_{\parallel}$, μ^{-1}	$R_{\perp}$	$R^b_{\parallel}$	$R^f_{\parallel}$	$R_{\perp}$	$R^b_{\parallel}$
1	1	1	1	0,33	0,33	0,33
1,5	2,6	1	0,38	1,0	0,38	0,15
5	9,9	1	0,10	5,3	0,54	0,05
10	19,9	1	0,05	12,7	0,64	0,03
50	$1\cdot10^2$	1	0,01	87,1	0,87	$8,7\cdot10^{-3}$
117	$2,3\cdot10^2$	1	$4,3\cdot10^{-3}$	$2,3\cdot10^2$	1,0	$4,3\cdot10^{-3}$
10^3	$2\cdot10^3$	1	$5\cdot10^{-4}$	$2,7\cdot10^3$	1,34	$6,7\cdot10^{-4}$
10^4	$2\cdot10^4$	1	$5\cdot10^{-5}$	$3,4\cdot10^4$	1,71	$8,6\cdot10^{-5}$
10^5	$2\cdot10^5$	1	$5\cdot10^{-6}$	$4,2\cdot10^5$	2,09	$1,0\cdot10^{-5}$

resent the nucleon "itself," this is simply tantamount to a growth in the dimensions of the nucleon, especially the longitudinal dimensions. One can say that the nucleon "swells." The ever greater elongation of the shape of the potential of the nuclear field in the forward direction with increasing γ is due to the "retardation factor" $\varkappa = 1 - \beta\cos\theta$. As we have already noted above, this is the reason for the analogous phenomenon for the Liénard–Wiechert equipotentials of the electromagnetic field of a moving charge. Thus, in both of these cases we are actually dealing with a manifestation what we have called the relativistic long-range field effect.

Details of the calculations are given in Table I. It can be seen that the growth in the range of the vector field is manifested in the first place along the axis of motion of the nucleon in the forward direction ($R^f_{\parallel}$). The transverse component $R^v_{\perp}$ increases much less strongly. For the pion field, $R^{\pi}_{\perp}$ does not change with increasing γ. At $\gamma \approx 10^2$, the nature of the nucleon field has been changed appreciably—the range of the vector field becomes comparable with R_{π}. With further growth of γ, the contribution of the vector field becomes dominant. As follows from Table I, at $\gamma = 10^3$ the longitudinal dimension of the nucleon is $2.7 \times 10^3\mu^{-1}$, while the transverse dimension is $1.34\mu^{-1}$. In addition, it is readily concluded that the angular dimension of the nuclear field (the ratio $R_{\perp}/R_{\parallel}$) decreases with increasing energy of the nucleon. This must necessarily have the consequence that in multiparticle production processes at high energies the emission angles of the secondary particles[9)] must be concentrated in a narrow cone with opening angle $\sim \gamma^{-1}$. Hadronic jets can serve as an example.

In our opinion, the fact that at $\gamma \gtrsim 10^2$ the dominant role will be played by the vector field of the nucleon warrants particular attention. This means that with increasing energy the contribution of interactions through vector mesons will increase. Thus, at high energies it is exchange of vector mesons that will characterize a peripheral collision. On the other hand, this must also obviously lead to a corresponding increase in the cross section for production of ρ and ω resonances, as indeed is observed experimentally (see, for example, Ref. 37). At the same time, since the ρ and ω mesons, in contrast to the pion, have spin, we must also expect a significant influence of spin effects as the energy is increased.

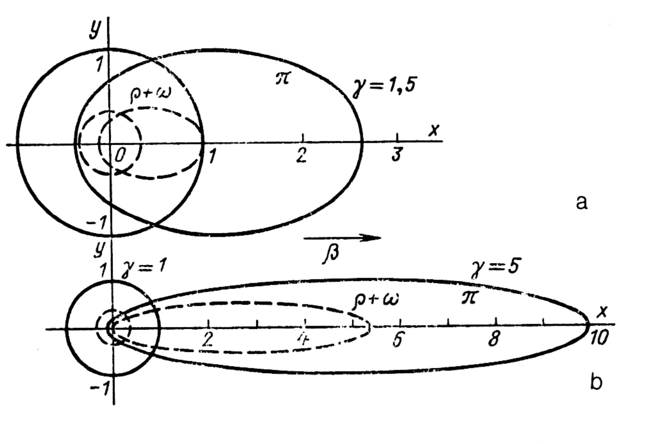

FIG. 5. Equipotential curves of the pion field and the field of the ρ and ω mesons (corresponding to the range of the nuclear forces) for $\beta = 0.745$ (a) and $\gamma = 0.980$ (b). The circles correspond to the ordinary Yukawa potential ($\beta = 0$).

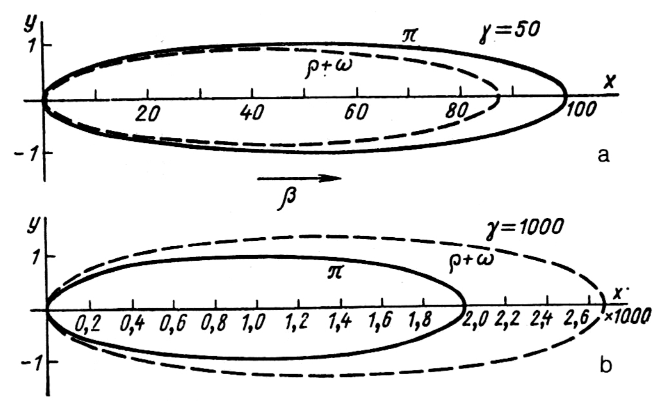

FIG. 6. The analogous equipotential curves of the pion field and the field of the ρ and ω mesons for $\beta = 0.999\ 79$ (a) and $\beta = 0.999\ 999\ 5$ (b).

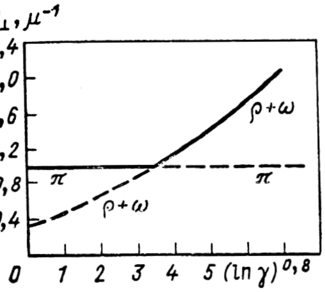

FIG. 7. Dependence of the change of the transverse nucleon dimension on the Lorentz factor.

However, the most important result may be that the transverse dimensions of the nucleon begin to increase by virtue of the vector field. As can be seen from Fig. 7, the variation of $R_\perp$ at large $\gamma \gtrsim 10^2$ can be fairly well described by the function $(\ln \gamma)^{0.8}$. This increase of the longitudinal dimensions must necessarily lead to a growth of the interaction cross sections, as is indeed observed experimentally (see, for example, Ref. 38). The analogous enhancement in the growth of the longitudinal dimension of the nucleon due to the field of the vector mesons must also lead to an enhancement in the growth of the formation length.

On the other hand, if it is borne in mind that the values of g_ρ and g_ω used above have a low accuracy, it can be seen that the change in the behavior of the total cross section could be used to determine them more accurately. Thus, proceeding from the condition that σ_{pp}^{tot} begins to increase at $\gamma = 90$, we obtain $g_v g_\pi^{-1} = 1$ instead of the previous value. On the basis of the more accurate values of the coupling constant from Ref. 39 we have $g_v = 0.96 \pm 0.08$.

Of course, these conclusions also hold for other baryons.

Duration of nuclear reactions

Nuclear reactions are transformations of nuclei as a result of interactions with elementary particles or with one another. A distinction is made between elastic scattering and direct processes, in which the energy introduced into a nucleus is transferred predominantly to one or a small group of nucleons. Finally, a compound nucleus that is unstable and decays after a certain time (its lifetime) may be formed. The different mechanisms of nuclear reactions are characterized by different times required for them to occur. The shortest time corresponds to a direct nuclear reaction (about 10^{-22} sec). The average lifetime of a compound nucleus is much greater (about 10^{-16}–10^{-15} sec). At high energies, direct processes are dominant. However, on the transition to high energies the contribution of reactions with production of resonance states increases.

At the present time, the concept of a nuclear reaction is rather broad and also covers interactions of the elementary particles themselves, for example, nucleons with nucleons, mesons with nucleons, etc.[10] Below, we shall actually consider nuclear reactions initiated by high-energy particles, and therefore we shall be basically considering multiparticle production processes. As already noted, in the overwhelming majority of cases these processes take place with the production of intermediate states—resonances.

We shall be interested in the way in which the duration of a nuclear reaction changes with the energy. The stimulus to the consideration of this question was the existing opinion that the duration decreases with increasing energy in proportion to γ^{-1}.

The assertion that the duration of nuclear reactions ($\tau_{\text{n.r}}$) decreases with increasing energy is actually based on the assumption that the longitudinal dimensions of a moving nucleus or other particle contract, i.e., that the dimensions are determined by the "synchronous length."[11] However, this takes no account at all of the relativistic effects of the field behavior, which is what essentially determined the mechanism of the nuclear reactions. Before we turn to this question, we wish to note the following. There is a certain analogy between the formation of a compound nucleus and the production of resonances. However, whereas the duration of nuclear reactions of the first type is effectively determined by the lifetime of the compound nucleus, in the second case it is the lifetime of the resonances. Because of the relativistic retardation of time, an increase in the velocity of the produced resonances leads to an increase of their lifetime and, therefore, of the duration of the corresponding reactions. Of course, it may be objected here that the duration of such nuclear reactions should be defined as that of the actual resonance production processes without the subsequent decay stages. However, the same objections can be applied to reactions with formation of a compound nucleus.

On the other hand, one can introduce a "field" definition of $\tau_{\text{n.r}}$. It appears entirely reasonable to say that the reaction commences at the time at which the field of the incident particle at the position of the test target particle (or an element of it) reaches a given value, and that the reaction ends at the time when its field at this point has decreased to the previous value after the passage of the particle. In this case, it is the behavior of the field of the relativistic particle that will determine the basic features of the development of nuclear reactions.

As was shown above, the longitudinal dimensions of the pion field of a nucleon increase in proportion to γ. Because the nucleon velocity is near unity, the reaction time (or the duration of the nuclear reaction) must also increase:

$$\tau_{\text{n.r}} \sim \gamma. \tag{41}$$

Bearing in mind the behavior of the nucleon vector field, we must expect an enhancement in the growth of $\tau_{\text{n.r}}$ at $\gamma \approx 10^2$.

We note also that in numerous nuclear reactions an incident nucleus or part of it interacts with the target as a whole. In other words, in the intrinsic system of the projectile its nucleons or some of them act simultaneously on the target. Examples of such reactions are elastic scattering, coherent particle production, cumulative processes, etc. One can say that in the reactions of this type the target will "see" the moving nucleus (or part of it) elongated in the direction of its motion in proportion to γ. Since at the same time the velocities are close to the speed of light, we again arrive at the qualitative conclusion that there is a growth of the interaction time.

4. SCALES OF LONGITUDINAL DISTANCES AT HIGH ENERGIES

The importance of large longitudinal distances in particle interactions at high energies can be regarded as something already somewhat familiar. The problem has been considered by numerous authors, and the results are summarized in the review of Ref. 40. Among the studies, we should perhaps distinguish Ref. 41, which raised the question of the distances at which the interactions occur in elastic and inelastic scattering (for example, of π mesons by nucleons) at high energies. It was pointed out that at high energies large longitudinal distances, which increase linearly with the energy of the incident particle, can play a part. Subsequent analysis of experimental data on electroproduction of hadrons on nucleons, photoproduction on protons, and absorption of neutrinos and antineutrinos by nucleons did indeed permit the conclusion to be drawn that large longitudinal distances play a dominant part.[42] In the study of inelastic interactions of fast hadrons with nuclei, it was concluded[43] that the longitudinal distances significant in hadron interactions increase as $\mathscr{E}M^{-2}$. Here, $\mathscr{E}$ is the hadron energy, and M is the nucleon mass.

All these rather different phenomena can be explained from a common point of view by assuming that the relevant longitudinal distances are simply determined by the formation lengths of the particles. Then the growth in the scales of the longitudinal distances at high energies will be a natural consequence of the increase of the formation length. It must be emphasized that this quantity plays an important part in both electromagnetic and strong interactions. Therefore, it can be regarded as a kind of connecting link that "unifies" these two types of interactions.

In view of the importance of this quantity, it is natural to wish to clarify more fully its physical meaning. For this, we must obviously consider the actual interaction event. Let us consider in particular the laboratory system, in which we have an incident particle, the projectile, a target at rest, and the produced secondary particles. According to modern ideas, these are all, in general, objects of finite sizes. It appears entirely reasonable to assume that the characteristic longitudinal scales of the interaction must be essentially determined by the longitudinal dimensions of its participants. Since the target is at rest, its contribution does not depend on the energy. There remain the incident particle and the reaction products. It appears natural that here the main contribution will be made by the object (particle) with maximal longitudinal dimension, this being ultimately determined by the field of the corresponding virtual quanta. Therefore, as already noted, the behavior of the field of a relativistic particle (be it an electron or a nucleon) considered above does indeed explain the corresponding behavior of the formation length and, therefore, the characteristic longitudinal dimensions.

On the other hand, large longitudinal distances are very difficult to reconcile with the traditional picture of relativistic objects as Lorentz-contracted disks. Since in this case the longitudinal dimensions of the incident and produced particles tend to zero, it would appear that the relevant distances must be determined by the longitudinal dimension of the target, i.e., remain constant with increasing energy.

Below, taking as examples models of elementary particles and the behavior of the spatial dimensions of the region of generation of identical pions, we shall present additional arguments for an increase of the longitudinal dimensions at high energies.

5. MODELS OF ELEMENTARY PARTICLES

The model of a relativistic string

Modern ideas about the structure of hadrons and the mechanism of their interaction at high energies led to the string model (see, for example, Ref. 44). In its framework, a relativistic string with point masses at the ends models the configurations of a gluon field localized along lines connecting quarks. A string connecting two quarks models a meson. Baryons, in the formation of which three quarks participate, can, for example, be described by a triangular string configuration, etc.

A string is a one-dimensional extended object. For hadronic physics, relativistic strings of finite size, or closed strings, have the greatest interest. Above all, the length of a string may change, and a string can even contract to a point. A closed string pulsates in time, periodically contracting to a point and then reverting to the original shape. It is important that, for example, the quarks at the ends of a string will reach their extreme positions simultaneously in the intrinsic system S^* of the string, in complete agreement with the expression (11) for the relativistic length. This is the qualitative picture. We now turn to a more detailed consideration.

Since electromagnetic fields act at large distances, one can say that the "retarded distances" discussed above (see Sec. 2) are macroscopic quantities. At the same time, there is an analogy between the electromagentic field and the gluon field. As is well known, the latter describes the behavior of the strong interactions at microscopic distances ($\lesssim 10^{-13}$ cm). The quanta of these fields, photons and gluons, are massless particles with spin 1. It is therefore supposed that the interaction between the quarks and gluons at short distances can be described by analogy with an electrostatic potential of the form $-\alpha_s/X$. However, there is here an important difference, namely, that the gluons have color charge and therefore interact with each other. The color forces draw the lines of force of the nuclear field into a filament. If the distance between the quarks is sufficiently large, this becomes a string, the tension in which is proportional to the distance between the quarks. The simplest hadron, the pion, is shown in Fig. 8. The potential that describes the main features of the interaction of color

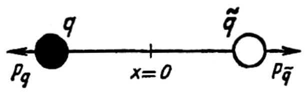

FIG. 8. String model of the pion.

charges can be written in the form (see, for example, Ref. 45)

$$U = -\alpha_s/X + kX. \quad (42)$$

In accordance with the well-known transition from the nonrelativistic Coulomb potential to the Liénard–Wiechert potential, we rewrite (42) in a relativistically covariant form:[46]

$$U^0 = (-\alpha_s/s + ks)u^0. \quad (43)$$

Here $s = u^i X_i$; u^i is the pion 4-velocity; X_i is the 4-vector of the retarded distance.

It follows necessarily from (43) that the dimension of a pion in motion, $l = X_{\tilde{q}} - X_q$, is related to its dimension l^* in the rest frame by an elongation formula. The value of l^* is determined by the extreme positions of the quark and antiquark ($-X^*$ and X^*, respectively) preceding, for example, the breaking of the string.

On the other hand, in accordance with Einstein's procedure for measurement in a moving system it is necessary to make simultaneous "soundings" of the positions of the quark and antiquark (the ends of the string). If we take, for example, the time at which the quark (the left end of the string) occupies the extreme position, then from the point of view of the system S^* these two events (soundings) will by no means correspond to the dimension of the pion at rest. Whereas for X_q^* we shall, as required, have $X_q^* = l^*/2$, for $X_{\tilde{q}}^*$ we find that $X_{\tilde{q}}^* \simeq l\gamma^{-2}/8$.[12)] Thus, we have arrived at an unphysical result. This is directly due to the fact that we actually ignored the retardation. Allowance for retardation means allowance for the existence of a limiting propagation velocity of interactions, and this is ultimately the basis of the theory of relativity and, therefore, of all relativistic theories.

Hydrodynamic model of multiparticle production

Investigations of hadronic matter, in particular, the new state of it represented by a quark–gluon plasma, are based on the construction of a space–time picture of collisions of ultrarelativistic nuclei and nucleons. At the present time, the only basis of such a picture is essentially the hydrodynamic theory of multiparticle production (see, for example, Ref. 47). On the other hand, this theory by itself, with small modifications, is claimed[48] to give a reasonable description of the most recent experimental data.

However, the hydrodynamic theory has a very serious shortcoming, namely, the classical initial conditions used in it, in the form of a Lorentz-contracted disk (resulting from the collision of two Lorentz-contracted nucleons or nuclei), contradict the quantum uncertainty principle.[49]

We recall that the hydrodynamic description presupposes interaction of a set of individual elements of the system. However, from the point of view of quantum theory such a division of the original system into individual elements (layers) is admissible only if the quantum uncertainty of the momentum is significantly less than the momentum of an individual layer. This requirement imposes a stringent restriction on the admissible number of layers of the Lorentz-contracted volume: $n \ll \sqrt{M/\mu} \simeq 2.6$, where M and μ are the masses of the nucleon and pion. With allowance for the additional (dynamical) compression due to the collision, this must apparently force us to abandon completely the hydrodynamic description of the initial stage of the expansion. It is true that recently there have been suggestions that the choice of initial Lorentz-contracted longitudinal dimensions contains a certain arbitrariness and could be modified, depending on various model representations (see, for example, Ref. 48, and also Ref. 50).

However, in our opinion a consistent and model-independent solution of this difficulty is possible only on the basis of the concept of the relativistic length.[51]

Indeed, in the framework of this concept moving nucleons are ellipsoids prolate in the direction of motion. In accordance with the "elongation formula," the major axis of the ellipsoid is

$$l = \mathscr{E}_c/\mu M, \quad (44)$$

where $\mathscr{E}_c$ is the energy of the nucleons. Restricting ourselves again to a one-dimensional problem, we now arrive at the condition

$$n \ll 2.6\ \gamma_c, \quad (45)$$

where $\gamma_c = \mathscr{E}_c/M$. Thus, at large γ_c this condition can no longer be satisfied, especially if we take into account the division in the transverse directions. On the other hand, it should be said that here we have not taken into account the dynamical compression due to the actual collision of the nucleons. Nevertheless, one way or the other, there is no doubt that it is the concept of the relativistic length that must provide the basis for the elimination of this difficulty, which, perhaps, it is more accurate to call a paradox, since, on the one hand, the hydrodynamic theory does describe certain features of multiparticle production processes while, on the other, it appears to be in crass contradiction with the uncertainty principle. In the light of what has been said, it now appears that the validity of the hydrodynamic theory can be regarded as an indirect argument in support of the concept of the relativistic length.

It is here necessary to emphasize the following. The picture usually sketched of colliding nucleons in the form of two Lorentz-contracted disks is an idealization. It is actually obtained by putting together two "photographs," each of which was taken at the instant at which the center of the nucleon was on the axis of the camera.[13)] But if the two nucleons are photographed at once (by a camera situated between them), then in the photograph they will be elongated, or certainly not Lorentz-contracted. However, we would here like to emphasize something different. The important thing is not how the nucleons "appear" to a spectator but how they "see each other." For simplicity, suppose that one nucleon moves and that the other is at rest. As we have noted, the process of "perception" is essentially the interaction with the target of, for example, photons or other field quanta radiated by the moving object. Thus, it is the behavior of the field of a relativistic nucleon, considered in Sec. 3, that gives the answer to our question. The incident nucleon has the shape of an ellipsoid of revolution prolate in the direction of motion. In this connection, it is interesting to note that, for example, the particles of galactic cosmic rays of the maximal energy (about 10^{21} eV) will actually have macroscopic longitudinal dimensions of about 1 cm.

Parton and Reggeon models

A disk picture of a fast hadron has usually been adopted in the framework of the parton model. However, this resulted in certain discrepancies between the consequences of the parton picture and the scheme of Reggeon diagrams.[52] To eliminate the difficulty, it was necessary to assume that a relativistic hadron is not a contracted disk but a tube of length $l \sim \mathscr{E} M^{-2}$. We note that the parton model is rather general and means simply that, for example, a nucleon is a composite object. According to modern ideas, partons are quarks and gluons, and these are, as it were, "unobservable parameters," since they have not yet been observed in the free state.

Below, we shall return to the parton model when considering the dipole moment of hadrons.

6. RELATIVISTIC DIPOLE MOMENT

Magnetic and electric dipole moments are important properties of material bodies and, in particular, of elementary particles; this applies especially to relativistic objects, i.e., essentially to the relativistic transformation of these moments. This question arose long ago (see, for example, Ref. 53). However, it appears that complete clarity has still not yet been achieved. A distinctive feature of the approach presented below is its manifestly relativistically covariant nature.

Transformation of dipole moment

It is well known that an intimate connection exists between the mechanical and the magnetic and electric properties of material bodies. Therefore, from the methodological point of view it may be better to begin our examination of this question with the transformation of the more fundamental angular momentum.

We recall that the relativistic angular momentum is determined by the time components of an antisymmetric 4-tensor M_{ik}:

$$M_{ik} = -\sum \varepsilon_{ikln} x^l p^n = \sum m\varepsilon_{ikln} x^l u^n. \qquad (46)$$

Here, x^l and $p^n = mu^n$ are the coordinate and 4-momentum of the particle; the summation is over all particles in the system. The spatial components $M_{\alpha\beta}$ describe the motion (position) of the center of mass of the system.

On the other hand, for the relativistic magnetic moment of a system of charges we have

$$\mathfrak{M}_{0\alpha} = -\frac{1}{2}\sum \frac{e}{m}\varepsilon_{0\alpha\beta\gamma} x^\beta p^\gamma = -\frac{1}{2}\sum e\varepsilon_{0\alpha\beta\gamma} x^\beta u^\gamma. \qquad (47)$$

The purely spatial components of the antisymmetric 4-tensor $\mathfrak{M}_{ik}$ determine the relativistic electric dipole moment.

If for all particles of the system the charge-to-mass ratio is the same, then e/m can be taken in front of the summation sign. As a result, we find that

$$\mathfrak{M}_{ik} = \frac{e}{2m} M_{ik}. \qquad (48)$$

Whereas the equation

$$\mathfrak{M}_{0\alpha} = \frac{e}{2m} M_{0\alpha} \qquad (48a)$$

describes the well-known connection between the magnetic moment and the mechanical momentum of the system, the analogous equation

$$\mathfrak{M}_{\alpha\beta} = \frac{e}{2m} M_{\alpha\beta} \qquad (48b)$$

describes the connection between the electric dipole moment and the position (motion) of the center of mass of the system.[14)]

With allowance for (48), and on the basis of the formulas for transforming the components of the mechanical M_{ik} (see, for example, Ref. 12), we can immediately write down the corresponding expressions for the relativistic dipole moment $\mathfrak{M}_{ik}$. Going over to familiar notation, i.e., setting

$$\mathfrak{M}_{0\alpha} = \mathfrak{M}_{x^\alpha}, \quad d_\alpha = \varepsilon_{\alpha\beta\gamma}\mathfrak{M}_{\beta\gamma}, \qquad (49)$$

where $\varepsilon_{\alpha\beta\gamma}$ is the completely antisymmetric unit tensor ($\varepsilon_{xyz} = -1$), we obtain

$$\mathfrak{M}_x = \mathfrak{M}_x^*; \qquad (50a)$$

$$\mathfrak{M}_y = (\mathfrak{M}_y^* + \beta d_z^*)\gamma; \qquad (50b)$$

$$\mathfrak{M}_z = (\mathfrak{M}_z^* - \beta d_y^*)\gamma; \qquad (50c)$$

$$d_x = d_x^*; \qquad (51a)$$

$$d_y = (d_y^* - \beta\mathfrak{M}_z^*)\gamma; \qquad (51b)$$

$$d_z = (d_z^* + \beta\mathfrak{M}_y^*)\gamma. \qquad (51c)$$

Here, the intrinsic frame of reference S^* of the collection of considered particles moves relative to the system S along the x axis with velocity β.

We now consider some special cases.

Loop with current

Suppose that the system of charges in which we are interested forms a loop with current for which the electric dipole moment vanishes: $\mathbf{d}=0$. If the loop with current is oriented at right angles to the direction of the motion, then on the basis of (50a) we conclude that, in contrast to the conclusions of Ref. 53, its magnetic moment is not changed on the transition from the system S^* to the system S. In contrast, the magnetic moment of a loop whose plane is parallel to the direction of motion increases on the basis of (50b) and (50c) with increasing velocity of the motion:

$$\mathfrak{M}_y=\mathfrak{M}_y^*\gamma,\quad \mathfrak{M}_z=\mathfrak{M}_z^*\gamma.$$

It can be seen that the last result also disagrees with the corresponding conclusion of Ref. 53.

With regard to the electric dipole moment, on the basis of (51a)–(51c) the longitudinal component d_x remains unchanged, while the transverse components increase with increasing velocity. Such behavior is again in disagreement with the conclusions of Ref. 53. Moreover, the assertion that the dipole moment $d_x^*=el^*$ of two charges e and $-e$ on the x^* axis at the points $x_e^*=l^*/2$ and $x_{-e}^*=-l^*/2$ does not change on the transition to the moving system appears altogether absurd, since the distance l^* must be transformed on the transition. The apparent paradox is due to the fact that the last expression is definitely nonrelativistic. In contrast, the corresponding relativistically covariant formula deduced from (47) has the form

$$d_x=-\sum\frac{e}{m}(tp_x-xp_t).\tag{52}$$

In the rest frame S^*, the momentum is $p_x^*=0$, the energy is $p_t^*=m$, and we do indeed obtain the familiar expression. However, in the moving system S, $p_x=m\beta\gamma$, $p_t=m\gamma$, and, with allowance for the transformation formulas for the coordinates, we find

$$d_x=-\frac{e}{m}[(\Delta t^*+\beta l^*)\gamma m\beta\gamma-(l^*+\beta\Delta t^*)\gamma m\gamma]=el^*.\tag{52a}$$

Thus, as we see, the longitudinal component of the electric dipole moment does indeed remain unchanged as a result of motion.

We consider in more detail the case when the loop with current (in the form of a square with sides l^*) lies in the plane (x^*, y^*). For simplicity, suppose that its center coincides with the origin and that the sides are parallel to the corresponding axes. The magnetic dipole is formed by four electrons that at the instant of the calculation are at the centers of the sides of the square. The current flows clockwise. We begin with the electron on the right edge of the square, for which $x^*=l^*/2$ (we then move counterclockwise). As a result, we find

$$\mathfrak{M}_z^*=\sum\frac{e}{2m}(x^*p_y^*-y^*p_x^*)$$

$$=\frac{-e}{2m}\left[\frac{l^*}{2}p_y-\frac{l^*}{2}(-p_x^*)+\left(-\frac{l^*}{2}\right)(-p_y^*)-\left(-\frac{l^*}{2}\right)p_x^*\right]$$

$$=\frac{-e}{m}l^*p^*,\tag{53}$$

where we have used the fact that $p_x^*=p_y^*=p^*$. Similarly, for d_y^*, for example, we have

$$-d_y^*=\sum\frac{e}{m}(y^*p_t^*-tp_y^*)$$

$$=\frac{-e}{m}\left[\frac{l^*}{2}m+\left(-\frac{l^*}{2}\right)m\right]=0.\tag{54}$$

The two terms given here correspond to the upper and lower electrons; for simplicity we have taken $t^*=0$. It must be emphasized that in the framework of relativity theory the times at which the right and left electrons are "taken" in the given case become important. Indeed, if $t_r^*\neq t_l^*$, we find that $d_y^*\neq 0$. However, in accordance with the ideas that have become established, the electric dipole moment of such a system must be zero. This means that the electrons must be "taken" simultaneously. However, the most important thing here may be that the last condition actually gives the transformation formula for the longitudinal dimension of the loop. Moreover, this implicitly introduces a definition of an important physical concept—the longitudinal dimension of a relativistically moving body. It is easy to see that since $t_r^*-t_l^*=0$ ("synchronicity condition"), the distance betwen the electrons in the moving system, $l=x_r-x_l$, must be determined by the "elongation formula" (3), where $l^*=x_r^*-x_l$.

Dipole moment of a hadron

In our opinion, the last example leads to important consequences for elementary-particle physics.[15)] According to modern ideas, hadrons are, for example, objects of finite size formed from quarks. In particular, the motion of the quarks leads to the existence of a magnetic moment. The neutron, say, consists of a u quark and two d quarks. In the simplest symmetric configuration, the u quark is at the center $(x^*=0)$, while the d quarks (with equal and opposite momenta) are at equal distances from the center (x^* and $-x^*$). Using the formula (52) for the dipole moment of the neutron, we find

$$d_x^*=\frac{e}{3m_d}(t_r^*-t_l^*)p_x^*.\tag{55}$$

Here, t_r^* and t_l^* are the time coordinates of the right and left d quarks. To ensure fulfillment of the experimental fact that there is no intrinsic electric dipole moment of the neutron, it is necessary to suppose that its constituents are "taken" simultaneously (synchronously) in the intrinsic frame S^* of the particles.[54] It is easy to show that this conclusion still holds for an asymmetric configuration of the quarks and for other hadrons. In other words, the interaction of a hadron with a target takes place in such a way that the quarks that constitute it act (on the average) simultaneously in the system S^*. The currently established upper limit for the intrinsic electric dipole moment of the neutron[55] enables us to conclude that the admissible nonsimultaneity for a quark velocity of about 1 cm/sec does not exceed 10^{-25} sec, while for velocities near the speed of light it does not exceed 10^{-35} sec. Thus, for a distance of order 1 cm the nonsimultaneity cannot be greater than 10^{-22} sec.

7. MEASUREMENT OF SPATIAL SIZE OF THE REGION OF GENERATION OF IDENTICAL PIONS IN DIFFERENT FRAMES OF REFERENCE

The well-known method of Hanbury-Brown and Twiss[56] for measuring the angular diameters of stars (so-called intensity interferometry) is based on the interference of photon pairs. The effect is determined by the square of the degree of coherence $|\gamma_{12}(\tau)|^2$, where τ is the path difference. For example, in the simplest case when the star is imaged by a circular disk with uniform radiation intensity

$$|\gamma_{12}| = \frac{2J_1(\tau)}{\tau}, \tag{56}$$

where J_1 is a Bessel function of the first kind.

In fact, a similar phenomenon—interference of pairs of different identical bosons, pions[16)]—was used at another time[57] to estimate the region of interaction in multiparticle production processes ($p\bar{p}$ annihilation). More recently, there have been numerous experiments of this type (see, for example, Ref. 58, in which other studies are cited). However, we shall be interested in only the studies of Refs. 59–61, in which the spatial dimensions of the region of generation of identical particles were determined relative to different frames of reference.

Usually, experimental data for pairs of identical pions with 4-momenta p_1^i and p_2^i have been analyzed on the basis of a formula for their density in phase space of the form[62–64]

$$W(p_1^i,p_2^i) = [1 + f(q)]\, W_{\text{ph}}(p_1^i,p_2^i), \tag{57}$$

where $W_{\text{ph}}(p_1^i,p_2^i)$ is the density of the pion pairs in the absence of interference. The function $f(q)$ was determined by the expression

$$f(q_0,q_T) = \frac{1}{1+(q_0 t)^2}\,|\gamma_{12}(q_T R)|^2 = f_1(q_0 t)\left[\frac{2J_1(q_T R)}{q_T R}\right]^2, \tag{58}$$

where $q_0 = p_1^0 - p_2^0$; $\mathbf{q}_T = \mathbf{q} - (\mathbf{q}n)\mathbf{n}$; $\mathbf{q} = \mathbf{p}_1 - \mathbf{p}_2$, $\mathbf{n} = (\mathbf{p}_1 + \mathbf{p}_2)/|\mathbf{p}_1 + \mathbf{p}_2|$; t is the lifetime of the sources; and R is the radius of the region of generation. Aspects of the theory of two-body interference correlations of identical pions were presented in the recently published review of Ref. 65.

In the experiments in which we are interested, a study was made of the interference effect for $\pi^-\pi^-$ pairs and of the total effect for $\pi^{\mp}\pi^{\mp}$ pairs. As the background distribution W_{ph}, the distribution of $\pi^+\pi^-$ pairs was taken. To estimate the influence of motion on the longitudinal dimensions of the generation region, pairs emitted in directions close to the normal relative to the reaction axis were selected.

We consider first investigations of multiparticle production processes accompanying the interaction of negative pions with nucleons at $p_{\pi^-} = 40$ GeV/c (Ref. 59) and with protons at $p_{\pi^-} = 5$ GeV/c (Ref. 60). These investigations found indications (strictly speaking, qualitative) that the longitudinal dimensions of the generation region increase on transition from the center-of-mass system to the laboratory coordinate system. If the intrinsic frame of reference (rest system) of the region of generation (S^*) is related to the center-of-mass system,[17)] then one can say that as a result of the motion there is an increase in the longitudinal dimensions of the region of generation.

Investigation of relativistic variation of spatial dimensions of the region of pion generation in $\tilde{p}p$ multiparticle-production reactions

As was already noted in Ref. 60, $\mathbf{q}_T$ is not a covariant quantity, and therefore its use, especially in work involving different frames of reference, requires some care. To satisfy the condition of covariance, i.e., that this quantity should describe the spatial components of a 4-vector, it is necessary to take a four-dimensional scalar product in the expression for $\mathbf{q}_T$. However, it is easy to show that this product vanishes, and the vector $\mathbf{q}$ is obviously covariant. On the other hand, it is $\mathbf{q}$ that determines the spatial part of the 4-vector whose time component is q_0. In fact, for $p_1^i \simeq p_2^i$, when the interference effect is actually observed, $\mathbf{q}$ and $\mathbf{q}_T$ are nearly equal. Nevertheless, as experiment shows, the effect is manifested more clearly in the variables $\mathbf{q}$. This is the reason why these variables were used to analyze the experimental data of Ref. 61 obtained when 22.4-GeV/c antiprotons interacted with protons.

The mean dimension of the interaction region was determined by fitting the q^2 distributions by the expression[18)]

$$g(q^2) \simeq a\left[1 + b\exp\left(-\frac{q^2R^2}{4}\right)\right], \tag{59}$$

where a and b are free parameters, with a correcting the normalization.

To determine the longitudinal and transverse dimensions of the pion emission region, the dependence of the interference effect on the orientation of the pairs with respect to the collision axis was studied. To estimate the influence of motion on the longitudinal dimension $R_\parallel$ of the interaction region, pairs satisfying the subsidiary conditions $|\cos\theta| < 1/\sqrt{2}$ and $|\cos\varphi| \geqslant 1/\sqrt{2}$ were selected. Here, θ is the angle between the direction n of the pair and the reaction axis, and φ is the angle between the plane of the pair and the plane (n, x). The transverse dimensions $R_\perp$ that were found corresponded to the condition $|\cos\theta| \geqslant 1/\sqrt{2}$. In addition, $\tilde{R}_\perp$ satisfying the requirement $|\cos\theta| < 1/\sqrt{2}$ were calculated. The Lorentz factor γ determined the frame of reference, with $\gamma = 1$ corresponding to the center-of-mass system, and $\gamma = 3.5$ corresponding to the laboratory system. Calculations were also made for intermediate values of γ and corresponding antilaboratory

TABLE II. Dependence of spatial dimensions of the region of generation on the Lorentz factor.

γ	$\langle R\rangle$, fm	$\langle R_\perp\rangle$	$\langle R_\perp\rangle$	$\langle R_\parallel\rangle$
1,0 (c.m.s.)	2,04±0,18	1,93±0,33	2,16±0,22	2,31±0,44
1,3	2,09±0,20	2,00±0,32	2,16±0,19	2,63±0,59 (1,78±0,34)
1,5	2,03±0,20	2,01±0,27	2,17±0,24	2,83±0,72 (1,54±0,29)
1,8	2,10±0,23	2,04±0,26	2,43±0,40	3,30±1,93 (1,28±0,24)
2,3	2,16±0,20	2,11±0,56	2,89±0,56	—
3,5 (lab. syst.)	2,07±0,30	1,95±0,29	—	—

systems, and the mean values were taken. Table II describes the dependence of the mean values of these quantites on the Lorentz factor. For $\langle R\rangle$ and $\langle \widetilde{R}_\perp\rangle$, the mean errors were also taken. The quantities $\widetilde{R}_\perp^2$, $R_\parallel^2$ and $\widetilde{R}_{\perp a}^2$, $R_{\parallel a}^2$, respectively, were regarded as statistically independent. To find $\langle R_\parallel^2\rangle$, the corresponding q^2 distributions were summed.

The values obtained in this way were smaller than the corresponding values of $R(q_T^2)$ usually calculated on the basis of the q_T^2 distributions [for example, in the center-of-mass system, $R(q_T^2) = 2.42 \pm 0.23$], and the interference effect itself is more pronounced in the variable q^2.

As can be seen from Table II, the spatial dimensions of the region of interaction and, in particular, its transverse dimensions, which make the main contribution to $\langle R\rangle$ with increasing γ, hardly change on the transition from the center-of-mass system to the laboratory system. At the same time, for $\widetilde{R}_\perp$, obtained by eliminating the pairs, which determine $R_\perp$, and for the longitudinal dimension a tendency is observed for these to increase with increasing Lorentz factor, admittedly with significant errors. This result agrees with analogous qualitative conclusions drawn in the papers mentioned above. Relating again the rest frame of the region of generation to the center-of-mass system,[19)] we can express this otherwise by saying that as a result of motion there is a relativistic increase in the longitudinal dimensions of the region. In accordance with the widely accepted opinion, motion should result in Lorentz contraction of rapidly moving objects. Therefore, for greater clarity Table II also gives, in brackets, the corresponding quantities calculated from $R_\parallel$ in the center-of-mass system on the basis of the Lorentz contraction formula.

The physical significance of these results was discussed in Ref. 60 with detailed use of an intuitive model of pion emission by the sources. Here, we only wish to note that experiments with high-energy particles provide more and more evidence that the relativistic dimensions are determined by sources that radiate ("flare up") simultaneously (or "on the average" simultaneously) in the intrinsic frame of reference. Essentially, the model is one further manifestation of the nonstandard concept of the relativistic length and, in particular, of its modification based on (11).

Of course, we emphasize once more that the results discussed here must, in view of the appreciable experimental errors, be determined more accurately, and they can be regarded only as qualitative indications. However, the most important thing here is, perhaps, that precisely these experiments have led to an understanding that the relativistic length must be manifested in multiparticle production processes. At the same time, the size of the generation region is measured by signals not in the form of light but in the form of pions.

Aspects of measurement of spatial dimensions of the region of generation of identical pions in different frames of reference[66]

As we have already noted, the correlation method is based on interference of second order, or intensity interferometry. In Refs. 59–61, interference of pairs of identical pions was investigated. If pions are emitted simultaneously by two sources separated by distance R, then, as is well known, the effect is determined by

$$I(\tau) \sim \cos \mathbf{qR}, \tag{60}$$

where $\mathbf{q} = \mathbf{p}_1 - \mathbf{p}_2$ is the difference of the pion momenta. Since the region of interference itself is actually characterized by small τ, we can replace (60) by the expression

$$I(\tau) \sim 1 - (1/2)\tau^2. \tag{60a}$$

In the experiments under discussion, we are dealing with the interaction of high-energy particles, or relativistic particles. In particular, the products of this interaction, the sources emitting the pions, can also move with relativistic velocities. Especially worthy of consideration here is the case when the entire interaction region (its center of mass) moves. In the general case, τ will be given by

$$\tau = \mathbf{qR} - q_0 t, \tag{61}$$

where q_0 is the difference of the pion energies, and t is the difference between the corresponding times of their generation.

For reactions in which identical particles collide, there is a distinguished, completely symmetric frame of reference—the center-of-mass system (S^*). As was noted some years ago,[60] in this system $t^* = 0$ on the average, at least for "symmetrically" emitted pions. There is an analogous symmetry (at least, kinematically, when one considers the interaction of particles of identical mass, for example, $\widetilde{p}p$ processes. We shall now give an analysis of experimental data relating to measurement of the spatial dimensions of the region of interaction relative to different frames of reference in such $\tilde{p}p$ reactions.[61] For this, we consider the laboratory system, in which the pion sources that form the interaction region move on the average with speed $v_x = \beta$.[20)] On the basis of Lorentz transformations and the symmetry condition noted above, we have

$$t = \beta R_x, \quad (62a)$$

$$R_x = R_x^*(1-\beta^2)^{-1/2}. \quad (62b)$$

After squaring of (61) in accordance with (60a), integration over the azimuthal angle (we have in mind axial symmetry of the reaction), and transition to the longitudinal, $R_\parallel \equiv R_x$, and transverse dimensions, we obtain, taking into account (62a),

$$\tau^2 \simeq q_x^2 R_\parallel^2 + q_\perp^2 R_\perp^2 - 2\beta q_x q_0 R_\parallel^2. \quad (63)$$

For simplicity, we have here omitted the term proportional to q_0^2 because it is small in the region of the interference peak. Under the condition $R_\parallel^2 \simeq R_\perp^2 \simeq R^2$, the preceding expression becomes

$$\tau^2 \simeq R^2(q^2 - 2\beta q_x q_0). \quad (63a)$$

It follows from this that to determine R^2 we must construct the distribution with respect to $Q = q^2 - 2\beta q_x q_0$. However, in practice the q^2 distributions are used. Therefore, for the corresponding R^2 we obtain overestimated or underestimated values, depending on the sign of the second term in (63a). This conclusion is still valid for the determination of $R_\perp^2$ and $R_\parallel^2$. Particularly important here is the fact that in the laboratory system and in the antilaboratory system corresponding to it the term just mentioned will make opposite contributions.[21)] To exclude its influence, it is obviously necessary to go over to the corresponding mean values.

All the features that we have listed were indeed brought out in the experiment considered above on the interference of identical pions in $\tilde{p}p$ interactions relative to different frames of reference.[61] The main aim of the experiment was to observe relativistic variation of the longitudinal dimension of the region of generation.

We recall that to determine the required $R_\parallel$ pairs of pions emitted at angle $\theta \approx \pi/2$ to the reaction axis were selected. In the laboratory system, these are the slowest pions, and for them we can use the nonrelativistic formula

$$q_{0tr} \simeq q_x u_x, \quad (64)$$

[22)]

where u_x is the mean pion velocity. Then on the basis of (63) we have

$$\tau_\parallel^2 \simeq R_\parallel^2 (1 - 2\beta u_x) q_x^2. \quad (65)$$

In the laboratory system, the pairs will, on the average, be displaced in the direction of the reaction axis, and therefore the values of $R_\parallel$ found on the basis of the q^2 distribution will be somewhat underestimated, while those found in the antilaboratory system will be accordingly overestimated. With regard to the transverse dimensions, they are mainly determined by fast π mesons emitted forward in the laboratory system. In this case, we can take for q_0 the ultrarelativistic expression

$$q_{0l} \simeq -\frac{q_x}{2u_x^2} - \frac{q_y}{2u_y^2} - \frac{q_z}{2u_z^2}. \quad (66)$$

Then taking, for example, for simplicity $q_y \approx -q_z$, we find

$$\tau_\perp^2 \simeq R_\perp^2 \left(1 + \beta \frac{q_x^2}{q_\perp^2 u_x^2} \frac{R_\parallel^2}{R_\perp^2}\right) q_\perp^2. \quad (67)$$

In other words, for $R_\perp$ overestimated values will be found experimentally in the laboratory system. For the reasons noted, such values will also have been observed in the experiment under consideration.

Thus, we now see that the transition to the mean values given in Table II has a justification, and the observed practical constancy of $\langle R_\perp \rangle$ with growth of the Lorentz factor only strengthens the arguments given above. With allowance for what we have said, the growth of the corresponding longitudinal dimensions of the region of generation of identical pions in pp interactions on the transition from the center-of-mass frame to the laboratory system can be regarded as an indication in support of the relativistic "elongation formula."

8. FUNDAMENTAL LENGTH AND THE RELATIVISTIC LENGTH[68]

The problem of a fundamental (or elementary) length has been discussed in various forms already for many years (see, for example, Refs. 69 and 70, and also Ref. 71). An elementary length has most often been introduced in connection with the problem of "divergences" in field theory. There exist several models of theories containing a fundamental length (unified theory of elementary particles, nonlocal quantum field theory, especially one of the most developed forms—the theory of quantized or discrete space–time, etc.). The problem of possible violations of causality in the microscopic world (violation of microcausality) is intimately related to the problem of a fundamental length. If a fundamental length l_0 does indeed exist, it is natural to suppose that it will play an important part in elementary-particle physics. It has been conjectured that the addition of l_0 to the two fundamental constants c and $\hbar$ would result in a complete basis, in terms of which a physical quantity of any dimension known to us could be expressed.

Although arguments for the existence of a fundamental length do not have the nature of rigorous assertions, the need for a reexamination of our ideas in the region of small space–time scales nevertheless appears very probable. According to modern estimates, $l_0 < 10^{-17}$–10^{-16} cm,[23)] though it is true, for example, that in grand unification theories one works with lengths of order 10^{-30}–10^{29} cm and right down to the gravitational (Planck) length $l_0 = \sqrt{\hbar G/c^3} \simeq 10^{-33}$ cm (G is the Newtonian gravitational constant).

At the same time, the fundamental problem of the limits of applicability of geometry (i.e., essentially macroscopic or classical notions) is still unanswered.

Thus, the introduction of a fundamental length means the introduction of some minimal length (minimal scale).[73] However, this step is in some contradiction with widely accepted ideas about relativistically moving rods. It was probably this that Pauli had in mind[74] when he commented that a universal minimal length could probably not exist on the basis of relativistic invariance.

Indeed, it is usually assumed that longitudinal dimensions must contract as a result of motion. But this means an unbounded decrease of l_0 with increasing velocity and, therefore, ultimately the impossibility of introducing an elementary length. At the same time, it is obvious that we do not have a similar difficulty in the case of the introduction of a minimal time interval.

In our opinion, a consistent solution of this contradiction is possible only on the basis of the concept of the relativistic length, in accordance with which longitudinal dimensions must increase as a result of motion. Therefore, in the framework of such a concept the introduction of a minimal spatial length will not lead to the occurrence of any such difficulties. In the light of what we have said, the arguments for the existence of a fundamental length can now be regarded as a further indirect argument in support of the concept of the relativistic length.

CONCLUSIONS

Thus, as we can see, the relativistic length plays a role of ever increasing importance in high-energy physics. This is due to the fact that, in essence, it reflects the space–time picture of the structure of elementary particles and the actual nature of their interaction. Since the relativistic length is expressed by the half-sum of two retarded distances, it is obvious that it is, from the very beginning, an organic consequence of electrodynamics.

We have established the following on the basis of the relativistic length (retarded distance).

On the basis of the Liénard–Wiechert potentials, it has been shown that the field of a relativistic charge is drawn out forward as the velocity of the charge is increased, and acts at ever greater distances ("long-range relativistic effect"). The growth of the formation length at high energies appears to be due to precisely this effect.

By means of the relativistic Yukawa potential, it has been established that a relativistic nucleon has the shape of an ellipsoid of revolution prolate in the direction of motion. With increasing energy, its pion and vector (ρ- and ω-meson) fields behave differently. The contribution of the vector field increases and at $\gamma \gtrsim 10^2$ becomes dominant, leading to a growth of the transverse nucleon dimensions and, therefore, to a growth of the interaction cross sections. The growth of the longitudinal dimensions must lead to an increase of the interaction time and to a growth of the formation lengths of hadrons. This is the main reason for a well-known phenomenon—the growth in the scales of longitudinal distances at high energies.

It has been demonstrated that the string and parton models and the hydrodynamic theory of multiparticle production either acquire a clear physical meaning on the basis of the concept of the relativistic length or are, essentially, based implicitly on it. In particular, application of this concept to the hydrodynamic model made it possible to eliminate a certain contradiction with the uncertainty principle.

On the basis of the expression for the relativistic electromagnetic dipole moment and the experimental fact that there is no intrinsic electric dipole moment of elementary particles the following has been shown. The interaction of, for example, a nucleon with a target occurs in such a way that the quarks that form it act simultaneously (in the intrinsic frame of the nucleon). This means that the longitudinal dimensions of the incident nucleon are described by the relativistic elongation formula.

Analysis of interference experiments to measure the spatial dimensions of the region of generation of identical pions in different frames of reference has enabled us to conclude that in them (at least, in the case of interaction of identical particles) the relativistic length is manifested.

Finally, it has been emphasized that the introduction of a fundamental length contradicts the widely accepted idea of a reduction of the longitudinal dimensions of rapidly moving objects, whereas in the framework of the concept of the relativistic length such a difficulty does not arise.

I thank G. N. Afanas'ev, É. G. Bubelev, V. A. Nikitin, M. S. Khvastunov, and, especially, V. M. Dubovik for helpful discussions.

[1)]The source moves in the direction of propagation of the field.
[2)]The charge moves away from the point of observation.
[3)]Somewhat symbolically, the English words "retarded" and "relativistic" begin with the same letter *r*.
[4)]The problem of the nonobservability of Lorentz contraction was first posed by Terrell.[16]
[5)]The significance of this last designation is that in accordance with (12) $l_r^0 \neq 0$, whereas in the framework of the generally adopted definition of the length of a moving rod $X_E^0 = 0$ ("synchronous formulation").
[6)]Here we have a complete analogy with the definition of perpendicularity in Euclidean geometry, which is covariant with respect to linear transformations.
[7)]This formula follows, in its turn, from the formula for the retarded potentials.
[8)]At the same time, it is understood that the relative velocity of the particles is still small.
[9)]The effect noted here can be interpreted as transformation of virtual quanta of the field into real particles.
[10)]Note, for example, that α decay also leads to transformation of nuclei, and therefore it should apparently be counted as a nuclear reaction.
[11)]The distance between simultaneous positions of the boundary points of the object.
[12)]With regard to the time of the "soundings" corresponding to the extreme position of the antiquark, in the case of breaking of the string it does not occur at all for the quark.
[13)]Strictly speaking, even this picture is not correct.
[14)]The inertial electric effect, the analog of the well-known Burnett effect, is a consequence of Eq. (48b).
[15)]Of course, in the case of elementary particles we must use the corresponding operators. However, the physical essence of the problem remains unchanged.
[16)]The effect was expressed in an excess of pairs of identical pions with small differences of their momenta.
[17)]This system is "distinguished" by its symmetry compared with other frames of reference. In this sense, reactions involving collisions of identical particles or, at least, particles of the same mass have special preference.
[18)]It should be noted that the numerical coefficient in the argument of the exponential depends on the shape of the region of generation and on the

positions of the sources. It determines the absolute value of R. Since we are interested in only the (relative) change of R on the transition from one frame of reference to another, its exact value is not important.

[19)] From the point of view of the kinematics, this already appears an entirely natural step.

[20)] In this connection, see also Ref. 67.

[21)] With regard to the omitted term, it will make a much smaller but constant contribution, and, in practice, this may lead to a slight overestimation of R^2.

[22)] We have omitted two other terms, which are small.

[23)] On the other hand, estimates of the upper limit for the electric dipole moment of the electron have yielded the much more stringent bound $l_0 \lesssim 10^{-24}$–10^{-23} cm (Ref. 72).

[1] H. Arzelies, Nuovo Cimento **35**, 783 (1965).

[2] F. Rohrlich, Nuovo Cimento **45B**, 76 (1966).

[3] G. Cavalleri and G. Salgarelli, Nuovo Cimento **62A**, 722 (1969).

[4] H. Ott, Z. Phys. **175**, 70 (1963).

[5] M. Planck, *Selected Works* [Russian translation] (Nauka, Moscow, 1975), p. 466.

[6] A. Einstein, Jahrb. Radioakt. Elektron. **4**, 411 (1907).

[7] C. Møller, *The Theory of Relativity*, 2nd ed. (Oxford University Press, London, 1972) [Russ. transl., Atomizdat, Moscow, 1975].

[8] A. Einstein, Ann. Phys. (N.Y.) **17**, 891 (1905).

[9] V. N. Strel'tsov, Communications R2-3482, R2-5555 [in Russian], JINR, Dubna (1967, 1971).

[10] V. N. Strel'tsov, Found. Phys. **6**, 293 (1976).

[11] A. Liénard, Eclairage Electr. **16**, 5 (1898); E. Wiechert, Arch. Neerl. **5**, 549 (1900).

[12] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields*, 4th English ed. (Pergamon, Oxford, 1975) [Russ. original, 6th ed., Nauka, Moscow, 1988].

[13] M. A. Duguay, Am. Sci. **59**, 551 (1971) [Russ. transl., Usp. Fiz. Nauk **109**, 157 (1973)].

[14] P. M. Mathews and M. Lakshmanan, Nuovo Cimento **B12**, 168 (1972).

[15] K. G. Suffern, Am. J. Phys. **56**, 729 (1988).

[16] J. Terrell, Phys. Rev. **116**, 1041 (1959).

[17] B. M. Bolotovskiĭ, in *Einstein Collection. 1980–1981* [in Russian] (Nauka, Moscow, 1985), p. 142.

[18] A. Gamba, Am. J. Phys. **35**, 83 (1967).

[19] S. Pahor and J. Strnad, Nuovo Cimento **20B**, 105 (1974).

[20] O. Gron, Am. J. Phys. **49**, 28 (1981).

[21] W. Burke, *Space-Time, Geometry, Cosmology* [Russ. transl., Mir, Moscow, 1985], p. 70.

[22] V. N. Strel'tsov, Communication R2-89-234 [in Russian], JINR, Dubna (1989).

[23] J. D. Jackson, *Classical Electrodynamics*, 2nd ed. (Wiley, New York, 1975) [Russ. transl. of 1st ed., Mir, Moscow, 1965].

[24] L. L. Nemenov, Yad. Fiz. **51**, 444 (1990) [Sov. J. Nucl. Phys. **51**, 284 (1990)].

[25] I. M. Frank, Izv. Akad. Nauk SSSR, Ser. Fiz. **6**, 3 (1942).

[26] B. M. Bolotovskiĭ, Tr. Fiz. Inst. Akad. Nauk SSSR **140**, 95 (1982).

[27] A. I. Akhiezer and N. F. Shul'ga, Usp. Fiz. Nauk **151**, 385 (1987) [Sov. Phys. Usp. **30**, 197 (1987)].

[28] W. Heitler, *The Quantum Theory of Radiation*, 3rd ed. (Clarendon, Oxford, 1954) [Russ. transl., IL, Moscow, 1956].

[29] A. Pais, in *Aspects of Quantum Theory*, edited by A. Salam and E. P. Wigner (Cambridge University Press, Cambridge, 1972), p. 79.

[30] V. N. Strel'tsov, Communication R2-6710 [in Russian], JINR, Dubna (1972).

[31] H. Poincaré, Rend. Circolo Matem. Palermo **21**, 129 (1906).

[32] B. Kwal, J. Phys. Radium **10**, 103 (1949).

[33] I. V. Polubarinov, Communication R2-7532 [in Russian], JINR, Dubna (1973).

[34] H. Yukawa, Proc. Phys. Mat. Soc. Jpn. **17**, 48 (1935).

[35] V. A. Belyakov and V. N. Stral'tsov, Preprint E2-90-309 [in English], JINR, Dubna (1990).

[36] G. E. Brown and A. D. Jackson, *The Nucleon–Nucleon Interaction* (North-Holland, Amsterdam, 1976) [Russ. transl., Atomizdat, Moscow, 1979].

[37] M. G. Albrow, S. Almehed, P. S. L. Booth *et al.*, Nucl. Phys. **B155**, 39 (1979).

[38] G. P. Yost, R. M. Barnett, I. Hinchliffe *et al.*, Phys. Lett. **204B**, 126 (1988).

[39] O. Dumbrajs, R. Koch, H. Pilkuhn *et al.*, Nucl. Phys. **B216**, 277 (1983).

[40] N. N. Nikolaev, Usp. Fiz. Nauk **134**, 369 (1981) [Sov. Phys. Usp. **24**, 531 (1981)].

[41] V. I. Gribov, B. L. Ioffe, and I. Ya. Pomeranchuk, Yad. Fiz. **2**, 768 (1965) [Sov. J. Nucl. Phys. **2**, 549 (1966)].

[42] B. L. Ioffe, Pis'ma Zh. Eksp. Teor. Fiz. **10**, 143 (1969) [JETP Lett. **10**, 90 (1969)].

[43] O. V. Kancheli, Pis'ma Zh. Eksp. Teor. Fiz. **18**, 465 (1973) [JETP Lett. **18**, 274 (1973)].

[44] B. M. Barbashov and V. V. Nesterenko, *Introduction to the Relativistic String Theory* (World Scientific, Singapore, 1989) [Russ. original, Énergoatomizdat, Moscow, 1987].

[45] V. S. Murzin and L. I. Sarycheva, *Physics of Hadronic Processes* [in Russian] (Énergoatomizdat, Moscow, 1986), p. 54.

[46] V. N. Strel'tsov, Communication R2-89-772 [in Russian], JINR, Dubna (1989).

[47] G. M. Zinov'ev, O. P. Pavlenko, and V. P. Shelest, in *Physics of Many-Particle Systems*, No. 10 [in Russian] (Naukova Dumka, Kiev, 1986), p. 65.

[48] I. L. Rozental' and Yu. A. Tarasov, Zh. Eksp. Teor. Fiz. **85**, 1535 (1983) [Sov. Phys. JETP **58**, 892 (1983)].

[49] D. I. Blokhintsev, Zh. Eksp. Teor. Fiz. **32**, 350 (1957) [Sov. Phys. JETP **5**, 286 (1957)].

[50] B. L. Ioffe, Pis'ma Zh. Eksp. Teor. Fiz. **20**, 360 (1974) [JETP Lett. **20**, 162 (1974)].

[51] V. N. Strel'tsov, Communication R2-87-617 [in Russian], JINR, Dubna (1987).

[52] O. V. Kancheli, Pis'ma Zh. Eksp. Teor. Fiz. **22**, 491 (1975) [JETP Lett. **22**, 237 (1975)].

[53] I. M. Frank, Usp. Fiz. Nauk **158**, 135 (1989) [Sov. Phys. Usp. **32**, 456 (1989)].

[54] V. N. Strel'tsov, Communication R2-88-173 [in Russian] JINR, Dubna (1988).

[55] K. F. Smith, N. Crampin, J. M. Bendlebury *et al.*, Phys. Lett. **234B**, 191 (1990).

[56] R. Hanbury-Brown and R. Q. Twiss, Philos. Mag. **45**, 663 (1954).

[57] G. Goldhaber, S. Goldhaber, W. Lee, and A. Pais, Phys. Rev. **120**, 300 (1960).

[58] A. Breakstone, R. Campanini, H. B. Crawley *et al.*, Phys. Lett. **162B**, 400 (1985).

[59] N. Angelov, N. Akhabyan, and V. G. Grishin, Yad. Fiz. **37**, 338 (1983) [Sov. J. Nucl. Phys. **37**, 202 (1983)].

[60] V. V. Glagolev, R. M. Lebedev, and V. N. Strel'tsov, Yad. Fiz. **42**, 181 (1985) [Sov. J. Nucl. Phys. **42**, 113 (1985)].

[61] V. N. Strel'tsov and V. V. Glagolev, Preprint R1-86-725 [in Russian], JINR, Dubna (1986).

[62] G. I. Kopylov and M. I. Podgoretskiĭ, Yad. Fiz. **18**, 656 (1973) [Sov. J. Nucl. Phys. **18**, 336 (1974)].

[63] G. I. Kopylov, Phys. Lett. **50B**, 472 (1974).

[64] G. Cocconi, Phys. Lett. **49B**, 459 (1974).

[65] M. I. Podgoretskiĭ, Fiz. Elem. Chastits At. Yadra **20**, 628 (1989) [Sov. J. Part. Nucl. **20**, 266 (1989)].

[66] V. N. Strel'tsov, Preprint R2-86-826 [in Russian], JINR, Dubna (1986).

[67] M. I. Podgoretskiĭ, Yad. Fiz. **37**, 455 (1983) [Sov. J. Nucl. Phys. **37**, 272 (1983)].

[68] V. N. Strel'tsov, Communication R2-88-626 [in Russian], JINR, Dubna (1988).

[69] D. I. Blokhintsev, *Space and Time in the Microworld* (Reidel, Dordrecht, 1973) [Russ. original, later edition, Nauka, Moscow, 1982].

[70] V. L. Ginzburg, *Physics and Astrophysics* (Pergamon, Oxford, 1985) [Russ. original, Nauka, Moscow, 1985].

[71] A. N. Vyal'tsev, *Discrete Space-Time* [in Russian] (Nauka, Moscow, 1965).

[72] V. G. Kadyshevskiĭ, Fiz. Elem. Chastits At. Yadra **11**, 5 (1980) [Sov. J. Part. Nucl. **11**, 1 (1980)].

[73] R. L. Liboff, Am. J. Phys. **55**, 1041 (1987).

[74] W. Pauli, "General principles of wave mechanics" [in German], in *Handbuch der Physik*, Vol. 24, Part 1, 2nd ed. (Berlin, 1933) [Russ. transl., Gostekhizdat, Moscow, 1947].

Translated by Julian B. Barbour